Steel Structures 7 (2007) 297-309

STEEL STRUCTURES
www.kssc.or.kr

Numerical Evaluation on Warping Constants of General Cold-Formed Steel Open Sections

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Abstract

The calculation of warping constant (C_w) for an open thin-walled open section is a tedious and difficult task and thus presenting an obstacle to routine design. Although C_w formulas and values for selective cold-formed steel sections are available in the AISI design manuals, most practicing engineers have limited idea of evaluating the warping constants for sections not listed in the AISI design manuals. This paper proposes a detailed step-by-step numerical procedure for evaluating the warping constant of a general open thin-walled section. Comparison shows virtually no difference between the calculated values and those listed in the AISI design manuals. The proposed procedure is a reliable and useful tool for computing the warping constant for an arbitrary cold-formed steel open section, which does not need a sophisticated computer software.

Keywords: warping constant, torsional stress, thin-walled section, cold-formed steel, torsional property

1. Introduction

Open thin-walled sections subjected to torsion will result in pure torsional shear stress, warping shear stress and warping normal stress. Evaluation of these torsion-induced stresses is not a routine job for practicing engineers as the major difficulty arises from the determination of sectional warping constant (C_w). Complex formulas for C_w usually involving mathematical integrations are available from previous researchers (Bleich, 1952; Galambos, 1968; Kollbrunner and Basler, 1969; Heins, 1975; Yu, 1991). Computation of warping constants for open thin-walled sections can be greatly simplified by recognizing the linear variation of unit warping properties between the two consecutive intersection points of plate elements. As a result, the sophisticated integral forms for C_w can be represented by numerical expressions suitable for computer coding as has been done in the past (Hancock 1978; Yoo and Acra, 1986; Kubo and Fukumoto, 1988; Trahair, 1993; Lue and Ellifritt, 1993, 2003; Salmon and Johnson, 1994; Papangelis and Hancock, 1995; Ellifritt and Lue, 1998; Lin and Hsiao, 2003). Although the C_w values and formulas for nine selective practical sections are provided in the AISI design manual (2002), the formulas presented are rather complicated to most users. When the open section is irregular or not available in the AISI Manual, a guide for computing C_w for such shapes is warranted. A simple and reliable approach for evaluating C_w values would be very useful for practicing engineers when dealing with torsional problems.

In this study, a step-by-step numerical procedure for C_w calculation is presented and discussed. A simple computer program was developed to facilitate the tedious calculation of C_w and a representative section was selected to illustrate the details of the proposed procedure. The accuracy of the proposed procedure was verified by comparing with the values tabulated in the AISI design manuals. The comparative study shows that the differences between the computed and tabulated C_w values are negligible. This implies that the proposed computational method to compute C_w for a cold-formed steel open section is accurate and reliable. The study will satisfy those who are looking for a better estimation on torsional evaluation of cold-formed steel open sections.

2. Integration Forms for Warping Constants of Open Thin-walled Sections

Integration forms for warping constants (C_w) of open thin-walled sections are summarized as follows, based on Galambos (1968) and Heins (1975). To calculate the C_w value, the centroid (C) and shear center (S) of a section must be located first. By basic mechanics, the centroid (C) coordinates of a section is given by Eq. (1).

$$X_c = \frac{1}{A}\int_A x\,dA \qquad Y_c = \frac{1}{A}\int_A y\,dA \tag{1}$$

where x and y are the coordinates of infinitesimal area dA along the section and A is the total area of open thin-walled section. The origin of the coordinate system is

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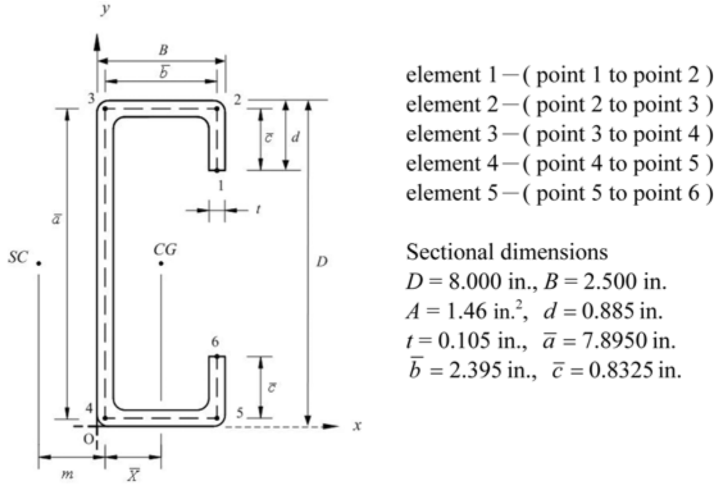

Figure 1. Section 8CS2.5 × 105 as used in the illustrative example.

usually set at the lower left corner (See Fig. 1).

According to Galambos (1968), the shear center (S) of a section is given by Eq. (2) with x and y as the principal centroidal axes (Figs. 2 and 3). The coordinates for the shear center of section is denoted by S (X_o, Y_o) with respect to the centroidal and principal x-y coordinate system.

$$X_o = \frac{1}{I_x}\int_o^E \rho\left(\int_o^S ytds\right)ds = \frac{1}{I_x}\int_o^E wytds = \frac{I_{wy}}{I_x} \tag{2}$$

$$Y_o = \frac{1}{I_y}\int_o^E \rho\left(\int_o^S xtds\right)ds = -\frac{1}{I_y}\int_o^E wxtds = -\frac{I_{wx}}{I_y} \tag{2a}$$

where

$$I_{wx} = \int_o^E wxtds \qquad I_{wy} = \int_o^E wytds \tag{2b}$$

In the above, I_x and I_y are moments of inertia about the x and y axes, respectively. The notation w implies the warping deformation of any point on the middle line at a distance s from the edge o (See Fig. 2). Hence, I_{wx} and I_{wy} represent the warping products of inertia about x- and y-axes, respectively.

The warping constant C_w of section can be determined by the following expression, Galambos (1968).

$$C_w = \int_o^E W_n^2 tds \tag{3}$$

where

$$W_n = \frac{1}{A}\int_o^E w_o tds - w_o \tag{3a}$$

$$w_o = \int_o^S \rho_o ds \tag{3b}$$

$$A = \int_o^E tds \tag{3c}$$

As shown above, W_n is the normalized unit warping, w_o the unit warping with respect to shear center, t the thickness of plate element, ρ_o the distance between the tangent and the shear center, and A is the area of section.

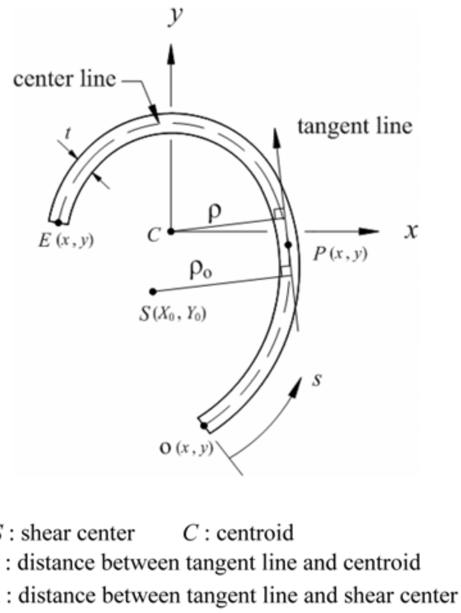

Figure 2. Coordinates and tangential distances in an open cross section.

3. Numerical Forms for Warping Constants of Open Thin-walled Sections

Since the open section is made up of thin plate elements, the computation of its warping constant may be greatly simplified by recognizing the linear variation of the unit warping properties (w, w_o, and W_n, see Fig. 4). The integration formulas of C_w proposed by Galambos (1968) can be rewritten in terms of numerical expressions as follows.

The centroid of section, C (X_c, Y_c):

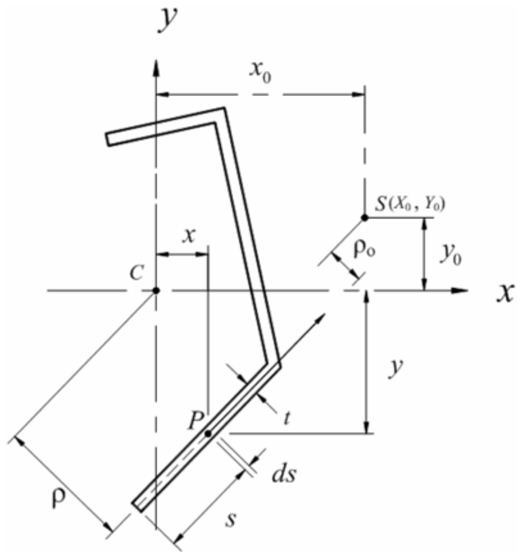

Figure 3. Coordinates and tangential distances in an open thin-walled section.

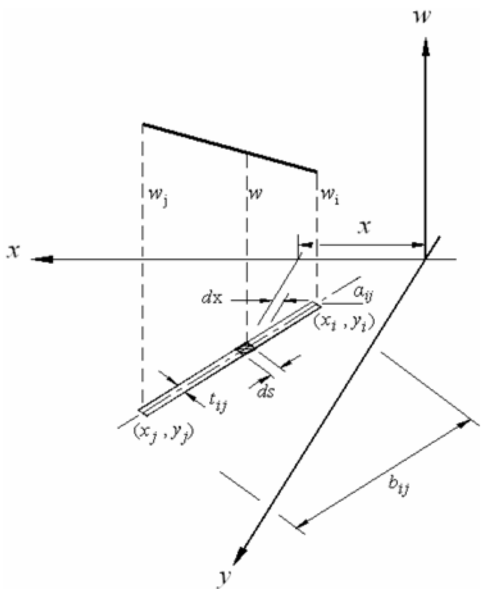

Figure 4. Distribution of w on a plate element.

$$X_c = \frac{\int_A x \ dA}{\int_A dA} = \frac{\sum_{i=1}^{n} x_i A_i}{\sum_{i=1}^{n} A_i}$$

$$Y_c = \frac{\int_A y \ dA}{\int_A dA} = \frac{\sum_{i=1}^{n} y_i A_i}{\sum_{i=1}^{n} A_i} \quad (4)$$

where A_i is the area of plate element i, and x_i and y_i are the x- and y-coordinates of plate element i, respectively (Fig. 1). Coordinates for the shear center of section (Fig. 3) measured from the centriod are given by:

$$X_o = \frac{I_{xy}I_{wx} - I_y I_{wy}}{I_{xy}^2 - I_x I_y} \qquad Y_o = \frac{I_x I_{wx} - I_{xy} I_{wy}}{I_{xy}^2 - I_x I_y} \quad (5)$$

where I_{xy} is the cross product of inertia of an area.

If the section is singly symmetrical being the case in the illustrative example, then $I_{xy} = 0$ and the above shear center equation can be simplified as:

$$X_o = \frac{I_{wy}}{I_x} \qquad Y_o = -\frac{I_{wx}}{I_y} \quad (6)$$

where

$$I_{wx} = \frac{1}{3}\sum_{i=1}^{n}(w_i x_i + w_j x_j) t_{ij} L_{ij} + \frac{1}{6}\sum_{i=1}^{n}(w_i x_j + w_j x_i) t_{ij} L_{ij} \quad (6a)$$

$$I_{wy} = \frac{1}{3}\sum_{i=1}^{n}(w_i y_i + w_j y_j) t_{ij} L_{ij} + \frac{1}{6}\sum_{i=1}^{n}(w_i y_j + w_j y_i) t_{ij} L_{ij} \quad (6b)$$

$$I_x = \int_A y^2 dA = \int_o^E y^2 t ds = \frac{1}{3}\sum_{i=1}^{n}(y_i^2 + y_i y_j + y_j^2) t_{ij} L_{ij} \quad (6c)$$

$$I_y = \int_A x^2 dA = \int_o^E x^2 t ds = \frac{1}{3}\sum_{i=1}^{n}(x_i^2 + x_i x_j + x_j^2) t_{ij} L_{ij} \quad (6d)$$

$$w_j = w_i + w_{ij} \qquad w_j = w_i + \rho_{ij} \times L_{ij} \qquad w_{ij} = \rho_{ij} \times L_{ij} \quad (6e)$$

In Eqs. (6a) to (6e), w is the unit warping with respect to the centroid, x_i, x_j, y_i and y_j the x and y coordinates at the ends of element, w_i and w_j the corresponding values of w at the ends of element, t the thickness of plate element, ρ_{ij} the distance between the tangent of element ij and the centroid, and L_{ij} is the length of the element ij.

The warping constant C_w can be given by:

$$C_w = \frac{1}{3}\sum_{i=1}^{n}(W_{ni}^2 + W_{ni}W_{nj} + W_{nj}^2) t_{ij} L_{ij} \quad (7)$$

where

$$W_{ni} = \left(\frac{1}{2A}\sum_{i=1}^{n}(w_{oi} + w_{oj}) t_{ij} L_{ij}\right) - w_{oi} \quad (7a)$$

$$W_{nj} = \left(\frac{1}{2A}\sum_{i=1}^{n}(w_{oi} + w_{oj}) t_{ij} L_{ij}\right) - w_{oj} \quad (7b)$$

$$w_{oj} = w_{oi} + \rho_{oij} \times L_{ij} \quad (7c)$$

In the above, w_o is the unit warping with respect to the shear center, W_n the normalized unit warping, w_{oi} and w_{oj} the corresponding values of w_o at the ends of element, t_{ij} the thickness of plate element ij, ρ_{oij} the distance between the tangent of element ij and the shear center, and L_{ij} is the length of element ij.

4. Numerical Example

The cold-formed C-section with lips, 8CS2.5×105, is used here to demonstrate the proposed procedure and the evaluation of sectional warping constant. The section is divided into 5 plate elements and their related joint numbers are as shown in Fig. 1 (D = 8.000 in., B = 2.500 in., t = 0.105 in., d = 0.885 in., A = 1.46 in.^{2}, $\bar{a}$ = 7.895 in., $\bar{b}$ = 2.395 in., and $\bar{c}$ = 0.8325 in., based on the 2002 AISI design manual).

The Proposed Step-by-Step Procedure:

(1) The centroid of section, (X_c, Y_c)

Coordinates of the centroid are referenced to the lower left corner being the origin of the coordinate system, as shown in Fig. 1. The coordinates and areas for each plate element (i) of the section are summarized in Table 1. Substituting Table 1 values (i.e., x_i , y_i , and A_i) into the formula for centroid Eq. (4) yields X_c = 0.73011 in. and Y_c = 4.00000 in.

(2) The shear center of section, (X_o, Y_o)

(a) Coordinates for the shear center are referenced to the centroid which can be determined by Eq. (6). The coordinates of element joints as measured from the centroid can therefore be computed as summarized in Table 2.

Table 1. Coordinates and areas of plate elements

Element no.(i)	x_i (in.)	y_i (in.)	A_i (in.^{2})
1	2.44750	7.53125	0.08741
2	1.25000	7.94750	0.25148
3	0.05250	4.00000	0.82898
4	1.25000	0.05250	0.25148
5	2.44750	0.46875	0.08741

x_i and y_i are coordinates with respect to lower left corner. 1 in. = 2.54 cm

(b) The moment inertias of a channel section with stiffened lips, taken with respect to the centroidal axes, calculated using Eqs. (6c) and (6d) are:

$$I_x = \int y^2 dA = 14.33383 \ in.^4 , \ I_y = \int x^2 dA = 1.27354 \ in.^4$$

(c) The warping products of inertia (I_{wx} and I_{wy}) are determined by Eqs. (6a) to (6e), in which the required parameters x_i , x_j , w_i , w_j , t_{ij} and L_{ij} are listed in Tables 2, 3, and 4. Substituting these parameters into Eqs. (6a) to (6e), results in:

$$I_{wx} = 0.00000 \ in.^5 \quad I_{wy} = -25.52550 \ in.^5$$

Detailed calculations for I_{wx} and I_{wy} are shown in Tables 5 and 6.

The shear center of section is then calculated using Eq. (6) with the known values of I_x , I_y , I_{wx} and I_{wy}. The shear center is therefore found to be at X_o = −1.78079 in. and Y_o = 0.00000 in. with respect to the centroid, or at X_s = −1.05068 in. and Y_s = 4.00000 in. with respect to the left lower corner of the section (See Fig. 1).

(3) The warping constant of section

The required parameters including w_{oi}, w_{oj}, W_{ni} and W_{nj} can be evaluated by using Eqs. (7a), (7b), and (7c), as summarized in Tables 7~9. Once W_n , W_{nj}, t_{ij} and L_{ij} values are calculated, the value of C_w can be computed by Eq. (7) which gives:

$$C_w = \frac{1}{3}\sum_{i=1}^{n}(W_{ni}^2 + W_{ni}W_{nj} + W_{nj}^2)t_{ij}L_{ij} = 16.693 \ in.^6$$

Details of C_w computation are demonstrated in Table 9. For convenience and comparison purposes, the C_w values for selective sections provided in the 2002 AISI design manual are provided in Tables 10~18.

Table 2. Joint coordinates, thicknesses, and lengths of plate elements

Element no.	Joint	x-coord. (in.)	y-coord. (in.)	t_{ij} (in.)	L_{ij} (in.)
1	1(i)	1.71739	3.11500	0.10500	0.83250
	2(j)	1.71739	3.94750		
2	2(i)	1.71739	3.94750	0.10500	2.39500
	3(j)	-0.67761	3.94750		
3	3(i)	-0.67761	3.94750	0.10500	7.89500
	4(j)	-0.67761	-3.94750		
4	4(i)	-0.67761	-3.94750	0.10500	2.39500
	5(j)	1.71739	-3.94750		
5	5(i)	1.71739	-3.94750	0.10500	0.83250
	6(j)	1.71739	-3.11500		

x_i and y_i coordinates with respect to the centroid of section. 1 in. = 2.54 cm

Table 3. Unit warpings w_i and w_j with respect to centroid of section

Element no.	ρ_{ij} (in.)	L_{ij} (in.)	$\rho_{ij}\times L_{ij}$	Joint	Unit warpings (in.^{2})	
					w_i	w_j
1	1.71739	0.83250	1.42973	1(i) 2(j)	0.00000 -	- 1.42973[a]
2	3.94750	2.39500	9.45426	2(i) 3(j)	1.42973 -	- 10.88399
3	0.67761	7.89500	5.34973	3(i) 4(j)	10.88399 -	- 16.23372
4	3.94750	2.39500	9.45426	4(i) 5(j)	16.23372 -	- 25.68798
5	1.71739	0.83250	1.42973	5(i) 6(j)	25.68798 -	- 27.11771

[a] $w_j = w_i + \gamma_{ij} \times L_{ij} = 0 + 1.42973 = 1.42973$ in.^{2} 1 in. = 2.54 cm

Table 4. Required data for evaluation of I_{wx} and I_{wy}

Element no.	Joint	Unit warpings (in.^{2})		x_i (in.)	x_j (in.)	y_i (in.)	y_j (in.)	t_{ij} (in.)	L_{ij} (in.)
		w_i	w_j						
1	1(i) 2(j)	0.00000 -	- 1.42973	1.717 -	- 1.717	3.115 -	- 3.948	0.10500	0.83250
2	2(i) 3(j)	1.42973 -	- 10.88399	1.717 -	- -0.678	3.948 -	- 3.948	0.10500	2.39500
3	3(i) 4(j)	10.88399 -	- 16.23372	-0.678 -	- -0.678	3.948 -	- -3.948	0.10500	7.89500
4	4(i) 5(j)	16.23372 -	- 25.68798	-0.678 -	- 1.717	-3.948 -	- -3.948	0.10500	2.39500
5	5(i) 6(j)	25.68798 -	- 27.11771	1.717 -	- 1.717	-3.948 -	- -3.115	0.10500	0.83250

Table 5. Calculation for warping product of inertia I_{wx}

	①	②	③	④	⑤	⑥	⑦
Element no.	w_ix_i	w_jx_j	$t_{ij}L_{ij}$	$\frac{1}{3}$(① + ②)③	w_ix_j	w_jx_i	$\frac{1}{6}$(⑤ + ⑥)③
1	0.000	2.455	0.0874	0.0715	0.000	2.455	0.0358
2	2.455	-7.375	0.2515	-0.4124	-0.969	18.692	0.7428
3	-7.375	-11.000	0.8290	-5.0775	-7.375	-11.000	-2.5388
4	-11.000	44.116	0.2515	2.7760	27.880	-17.406	0.4390
5	44.116	46.572	0.0874	2.6424	44.116	46.572	1.3212
Σ				0.0000			0.0000
					I_{wx} = 0.0000 in.^{5} [a]		

[a] $I_{wx} = \frac{1}{3}\sum_{i=1}^{n}(w_ix_i + w_jx_j)t_{ij}L_{ij} + \frac{1}{6}\sum_{i=1}^{n}(w_ix_j + w_jx_i)t_{ij}L_{ij}$

5. Results of Comparison between the Calculated and the Listed Values

The warping constant, C_w = 16.693 in.^{6}, for the channel section with lips, 8CS2.5×105, was obtained using the proposed computational procedure with $\alpha = 1$ ($\alpha = 1$ section with stiffener lips, $\alpha = 0$ section without stiffener lips). This computed C_w value matches with that contained in the 2002 AISI design manual. The relatively new C_w formula, Eq. (10), for C-sections with lips was

Table 6. Calculation for warping product of inertia I_{wy}

	①	②	③	④	⑤	⑥	⑦
Element no.	$w_i y_i$	$w_j y_j$	$t_{ij} L_{ij}$	$\frac{1}{3}$(① + ②)③	$w_i y_j$	$w_j y_i$	$\frac{1}{6}$(⑤ + ⑥)③
1	0.000	5.644	0.0874	0.1644	0.000	4.454	0.0649
2	5.644	42.965	0.2515	4.0746	5.644	42.965	2.0373
3	42.965	-64.083	0.8290	-5.8354	-42.965	64.083	2.9177
4	-64.083	-101.403	0.2515	-13.8719	-64.083	-101.403	-6.9359
5	-101.403	-84.472	0.0874	-5.4159	-80.018	-107.047	-2.7253
Σ				-20.8842			-4.6413
					$I_{wy} = -25.5255$ in.^{5} [a]		

[a] $I_{wy} = \frac{1}{3}\sum_{i=1}^{n}(w_i y_i + w_j y_j) t_{ij} L_{ij} + \frac{1}{6}\sum_{i=1}^{n}(w_i y_j + w_j y_i) t_{ij} L_{ij}$

Table 7. Unit warpings w_{oi} and w_{oj} with respect to shear center

Element no.	ρ_{oij} (in.)	L_{ij} (in.)	$\rho_{oij}\ L_{ij}$ (in.^{2})	Joint	Unit warpings (in.^{2})	
					w_{oi}	w_{oj}
1	3.49818	0.8325	2.91223	1(i) 2(j)	0.00000 -	- 2.91223[a]
2	3.94750	2.3950	9.45426	2(i) 3(j)	2.91223 -	- 12.36650
3	-1.10318	7.8950	-8.70959	3(i) 4(j)	12.36650 -	- 3.65690
4	3.94750	2.3950	9.45426	4(i) 5(j)	3.65690 -	- 13.1117
5	3.49818	0.8325	2.91223	5(i) 6(j)	13.11117 -	- 16.02340

[a] $w_{oj} = w_{oi} + \rho_{oij} \times L_{ij} = 0 + 2.91223 = 2.91223$ in.^{2}

Table 8. Normalized unit warpings W_{ni} and W_{nj}

Element no.	Joint	w_{oi} (in.^{2})	w_{oj} (in.^{2})	$t_{ij} L_{ij}$ (in.^{2})	W_{ni} (in.^{2})	W_{nj} (in.^{2})
1	1(i) 2(j)	0.00000 -	- 2.91223	0.0874	8.01170 -	- 5.09947
2	2(i) 3(j)	2.91223 -	- 12.36650	0.2515	5.09947 -	- -4.35480
3	3(i) 4(j)	12.36650 -	- 3.65690	0.8290	-4.35480 -	- 4.35480
4	4(i) 5(j)	3.65690 -	- 13.1117	0.2515	4.35480 -	- -5.09947
5	5(i) 6(j)	13.11117 -	- 16.02340	0.0874	-5.09947 -	- -8.01170

developed based on the proposed procedure and involves sectional dimensions only. While the AISI Manual (1991) requires more information including cross-sectional area (A), the distance between shear center and web centerline (m), and moment of inertia (I_x).

Based on the comparative study, there are virtually no differences between the calculated and AISI listed C_w values. This proves that the proposed computational procedure for C_w is accurate and reliable. Furthermore, the proposed procedure and formulas are more suitable for practical use.

Table 9. Calculation for warping constant of C-sections with lips

Element no.	Joint	W_{ni}^2 (in.[4])	$W_{ni}\times W_{nj}$ (in.[4])	W_{nj}^2 (in.[4])	$t_{ij}L_{ij}$ (in.[2])	$(C_w)_i$ (in.[6])
1	1(*i*) 2(*j*)	64.1873 -	40.8554	- 26.0046	0.0874	3.8184 [a]
2	2(*i*) 3(*j*)	26.0046 -	-22.2071	- 18.9643	0.2515	1.9080
3	3(*i*) 4(*j*)	18.9643 -	-18.9643	- 18.9643	0.8290	5.2403
4	4(*i*) 5(*j*)	18.9643 -	-22.2071	- 26.0046	0.2515	1.9080
5	5(*i*) 6(*j*)	26.0046 -	40.8554	- 64.1873	0.0874	3.8184
Σ						16.693 [b]

a. $(C_w)_i=(W_{ni}^2+W_{ni}W_{nj}+W_{nj}^2)t_{ij}L_{ij}/3=(64.187+40.855+26.005)\times 0.0874/3=3.818\,in.^6$

b. $C_w=\sum_{i=1}^{n}(C_w)_i=16.693\,in.^6$

Table 10. Comparisons of C_w values for selective C-sections without lips

Section	*D* (in.)	*B* (in.)	*t* (in.)	*A* (in.[2])	C_w (in.[6]) calculated by authors	C_w (in.[6]) provided by AISI manual
1200T200-68	12.250	2.00	0.0713	1.140	8.43100	8.4300
1000T125-97	10.356	1.25	0.1017	1.270	2.12200	2.1200
800T150-43	8.161	1.50	0.0451	0.496	0.97150	0.9720
550T125-54	5.698	1.25	0.0566	0.452	0.31542	0.3150
400T125-30	4.141	1.25	0.0312	0.203	0.08552	0.0855
350T125-18	3.622	1.25	0.0188	0.113	0.03842	0.0384

See Figure 1 for the notations.

Table 11. Comparisons of C_w values for selective C-sections with lips

Section	*D* (in.)	*B* (in.)	*t* (in.)	*A* (in.[2])	C_w (in.[6]) calculated by authors	C_w (in.[6]) provided by AISI manual
12CS4×070	12.000	4.00	0.070	1.47	80.763	80.8
10CS4×105	10.000	4.00	0.105	1.99	81.736	81.7
10CS2×085	10.000	2.00	0.085	1.27	12.996	13.0
9CS2.5×059	9.000	2.50	0.059	0.881	11.905	11.9
8CS3.5×070	8.000	3.50	0.070	1.12	24.098	24.1
7CS4×105	7.000	4.00	0.105	1.67	38.092	38.1

See Figure 1 for the notations.

6. Summary of the Derived and the Listed C_w Formulas

There are nine selective practical sections contained in the AISI design manuals (1991, 1996, 2002), as shown in Fig. 5. The derived formulas expressed in terms of sectional dimensions are obtained according to the proposed procedure. For comparison purpose, the derived and the AISI listed formulas are summarized as follows.

Warping constant for C-sections without lips:

$$C_w=\frac{\bar{b}^3t\bar{a}^2}{12}\left(\frac{3\bar{b}+2\bar{a}}{\bar{a}+6\bar{b}}\right)$$

(From this study and 1991 AISI Manual) (8)

Table 12. Comparisons of C_w values for selective Z-sections without lips

Section	D (in.)	B (in.)	t (in.)	A (in.^{2})	C_w (in.^{6}) calculated by authors	C_w (in.^{6}) provided by AISI manual
8ZU1.25×105	8.000	1.25	0.105	1.06	1.54600	1.5500
6ZU1.25×075	6.000	1.25	0.075	0.612	0.61180	0.6120
4ZU1.25×048	4.000	1.25	0.048	0.299	0.16410	0.1640
3.625ZU1.25×060	3.625	1.25	0.060	0.349	0.16045	0.1610
2.5ZU1.25×090	2.500	1.25	0.090	0.416	0.09527	0.0953
1.5ZU1.25×036	1.500	1.25	0.036	0.135	0.01273	0.0127

See Figure 1 for the notations.

Table 13. Comparisons of C_w values for selective Z-sections with lips

Section	D (in.)	B (in.)	t (in.)	A (in.^{2})	C_w (in.^{6}) calculated by authors	C_w (in.^{6}) provided by AISI manual
12ZS3.25×085	12	3.25	0.085	1.70	98.874	98.60
10ZS2.75×070	10	2.75	0.070	1.19	37.536	37.50
9ZS2.25×105	9	2.25	0.105	1.57	29.668	29.70
8ZS3.25×065	8	3.25	0.065	1.04	30.470	30.50
7ZS2.25×059	7	2.25	0.059	0.763	9.456	9.46
4ZS2.25×070	4	2.25	0.070	0.696	3.383	3.38

See Figure 1 for the notations.

Table 14. Comparisons of C_w values for hat sections without lips

Section	D (in.)	B (in.)	t (in.)	A (in.^{2})	C_w (in.^{6}) calculated by authors	C_w (in.^{6}) provided by AISI manual
10HU5×075	10.000	5.00	0.075	1.98	98.410	98.40
8HU8×105	8.000	8.00	0.105	2.71	222.060	222.00
6HU3×060	6.000	3.00	0.060	0.954	6.039	6.04
4HU4×105	4.000	4.00	0.105	1.45	6.893	6.89
3HU4.5×135	3.000	4.50	0.135	1.74	5.655	5.66
3HU3×075	3.000	3.00	0.075	0.761	1.143	1.14

See Figure 1 for the notations.

Table 15. Comparisons of C_w values for equal leg angles with lips

Section	D (in.)	B (in.)	t (in.)	A (in.^{2})	C_w (in.^{6}) calculated by authors	C_w (in.^{6}) provided by AISI manual
4LS4×135	4.000	4.000	0.135	1.120	0.09072	0.0907
4LS4×075	4.000	4.000	0.075	0.636	0.06314	0.0631
3LS×105	3.000	3.000	0.105	0.669	0.04203	0.0420
3LS×060	3.000	3.000	0.060	0.392	0.02857	0.0286
2.5LS2.5×090	2.500	2.500	0.090	0.489	0.02556	0.0256
2LS2×135	2.000	2.000	0.135	0.576	0.01922	0.0192

See Figure 1 for the notations.

Table 16. Comparisons of C_w values for equal leg angles without lips

Section	D (in.)	B (in.)	t (in.)	A (in.^{2})	C_w (in.^{6}) calculated by authors	C_w (in.^{6}) provided by AISI manual
4LU4×135	4.000	4.000	0.135	1.047	0.000	0.000
4LU4×090	4.000	4.000	0.090	0.703	0.000	0.000
3LU3×135	3.000	3.000	0.135	0.777	0.000	0.000
3LU3×090	3.000	3.000	0.090	0.523	0.000	0.000
2.5LU2.5×105	2.500	2.500	0.105	0.503	0.000	0.000
2LU2×135	2.000	2.000	0.135	0.507	0.000	0.000

See Figure 1 for the notations.

Table 17. Comparisons of C_w values for selective double channels with unstiffened flanges back-to-back sections in 1991 AISI Manual

Section	D (in.)	B (in.)	t (in.)	A (in.^{2})	C_w (in.^{6}) calculated by authors	C_w (in.^{6}) provided by AISI manual
2-7C3×135	7.0	3.00	0.135	2.568	6.2300	6.2300
2-7C3×105	7.0	3.00	0.105	2.013	5.0500	5.0500
2-6C3×135	6.0	3.00	0.135	2.298	4.5500	4.5500
2-6C3×105	6.0	3.00	0.105	1.803	3.6900	3.6900
2-3C2.25×105	3.0	2.25	0.105	1.015	0.3620	0.3620
2-2C2.25×105	2.0	2.25	0.105	0.805	0.1550	0.1550

See Figure 1 for the notations.

Table 18. Comparisons of C_w values for selective double channels with stiffened flanges back-to-back sections in 1991 AISI Manual

Section	D (in.)	B (in.)	t (in.)	A (in.^{2})	C_w (in.^{6}) calculated by authors	C_w (in.^{6}) provided by AISI manual
2-12CS7×135	12.0	7.00	0.135	5.411	478.00	478.00
2-8CS6×135	8.0	6.00	0.135	4.018	138.00	138.00
2-7CS5.5×135	7.0	5.50	0.135	3.586	82.30	82.30
2-5CS4×135	5.0	4.00	0.135	2.544	15.90	15.90
2-4CS4×135	4.0	4.00	0.135	2.274	10.40	10.40
2-3.5CS4×135	3.5	4.00	0.135	2.139	8.09	8.09

See Figure 1 for the notations.

$$C_w = \frac{\bar{a}^2\bar{b}^2 t}{12}\left\{\frac{2\bar{a}^3\bar{b}+3\bar{a}^2\bar{b}^2+\alpha\begin{bmatrix}48\bar{c}^4+112\overline{bc}^3+8\overline{ac}^3+48\overline{abc}^2\\ +12\bar{a}^2\bar{c}^2+12\bar{a}^2\overline{bc}+6\bar{a}^3\bar{c}\end{bmatrix}}{6\bar{a}^2\bar{b}+(\bar{a}+\alpha 2\bar{c})^3-\alpha 24\overline{ac}^2}\right\}$$

where $\alpha = 0$ (From 1996 and 2002 AISI Manual) (9)

Warping constant for C-sections with lips:

$$C_w = \frac{\bar{a}^2\bar{b}^2 t}{12}\left\{\frac{2\bar{a}^3\bar{b}+3\bar{a}^2\bar{b}^2+\alpha\begin{bmatrix}48\bar{c}^4+112\overline{bc}^3+8\overline{ac}^3+48\overline{abc}^2\\ +12\bar{a}^2\bar{c}^2+12\bar{a}^2\overline{bc}+6\bar{a}^3\bar{c}\end{bmatrix}}{6\bar{a}^2\bar{b}+(\bar{a}+\alpha 2\bar{c})^3-\alpha 24\overline{ac}^2}\right\}$$

where $\alpha = 1$ (From this study and 1996, 2002 AISI Manuals) (10)

$$C_w=\frac{t^2}{A}\left\{\begin{array}{l}\frac{\bar{x}A\bar{a}^2}{t}\left[\frac{\bar{b}^2}{3}+m^2-m\bar{b}\right]+\frac{A}{3t}[m^2\bar{a}^3+\bar{b}^2c^2 2\bar{c}+3\bar{a}]-\frac{I_x m^2}{t}(2\bar{a}+4\bar{c})+\\ \frac{m\bar{c}^2}{3}[8\bar{b}^2\bar{c}+2m(2\bar{c}(\bar{c}-\bar{a})+\bar{b}(2\bar{c}-3\bar{a}))]+\frac{\bar{b}^2\bar{a}^2}{6}[(3\bar{c}+\bar{b}^2)(4\bar{c}+\bar{a})-6\bar{c}^2]-\frac{m^2\bar{a}^4}{4}\end{array}\right\}$$

(From 1991 AISI Manual) (11)

Warping constant for Z-sections without lips:

$$C_w=\frac{\bar{b}t\bar{a}^2}{12}\left(\frac{\bar{b}+2\bar{a}}{\bar{a}+2\bar{b}}\right)$$ (From this study) (12)

$$C_w=t\bar{b}^2\left\lfloor\bar{a}^2(\bar{b}^2+2\overline{ab}+4\alpha\overline{bc}+6\overline{ac})+4\alpha\bar{c}^2(3\bar{a}^2+3\overline{ab}+4\overline{bc}+2\overline{ac}+\bar{c}^2)\right\rfloor/12(\bar{a}+2\bar{b}+2\bar{c})$$

where $\alpha = 0$ (From 1991 AISI Manual) (13)

$$C_w=\frac{t}{12}\left\{\frac{\bar{a}^2\bar{b}^3(2\bar{a}+\bar{b})+\alpha\left[\begin{array}{l}\bar{b}^2(4\bar{c}^4+16\overline{bc}^3+6\bar{a}^3\bar{c}+4\bar{a}^2\overline{bc}+8\overline{ac}^3)\\ +6\overline{abc}^2(\bar{a}+\bar{b})(2\bar{b}\sin\gamma+\bar{a}\cos\gamma)\\ +4\overline{abc}^3(2\bar{a}+4\bar{b}+\bar{c})\sin\gamma\cos\gamma\\ +\bar{c}^3(2\bar{a}^3+4\bar{a}^2\bar{b}-8\overline{ab}^2+\bar{a}^2\bar{c}-16\bar{b}^3-4\bar{b}^2\bar{c})\cos^2\gamma\end{array}\right]}{\bar{a}+2(\bar{b}+\alpha\bar{c})}\right\}$$

where $\alpha = 0$ (From 1996 and 2002 AISI Manuals) (14)

Warping constant for Z-sections with lips:

$$C_w=\frac{t}{12}\left\{\frac{\bar{a}^2\bar{b}^3(2\bar{a}+\bar{b})+\left[\begin{array}{l}\bar{b}^2(4\bar{c}^4+16\overline{bc}^3+6\bar{a}^3\bar{c}+4\bar{a}^2\overline{bc}+8\overline{ac}^3)\\ +6\overline{abc}^2(\bar{a}+\bar{b})(2\bar{b}\sin\gamma+\bar{a}\cos\gamma)\\ +4\overline{abc}^3(2\bar{a}+4\bar{b}+\bar{c})\sin\gamma\cos\gamma\\ +\bar{c}^3(2\bar{a}^3+4\bar{a}^2\bar{b}-8\overline{ab}^2+\bar{a}^2\bar{c}-16\bar{b}^3-4\bar{b}^2\bar{c})\cos^2\gamma\end{array}\right]}{\bar{a}+2(\bar{b}+\bar{c})}\right\}$$

where $\gamma = 50^\circ$ (From this study) (15)

$$C_w=t\bar{b}^2[\bar{a}^2(\bar{b}^2+2\overline{ab}+4\alpha\overline{bc}+6\overline{ac})+4\alpha\bar{c}^2(3\bar{a}^2+3\overline{ab}+4\overline{bc}+2\overline{ac}+\bar{c}^2)]/12(\bar{a}+2\bar{b}+2\bar{c})$$

where $\alpha = 1$, $\gamma = 90^\circ$ (From 1991 AISI Manual) (16)

$$C_w=\frac{t}{12}\left\{\frac{\bar{a}^2\bar{b}^3(2\bar{a}+\bar{b})+\alpha\begin{bmatrix}\bar{b}^2(4\bar{c}^4+16\bar{b}\bar{c}^3+6\bar{a}^3\bar{c}+4\bar{a}^2\bar{b}\bar{c}+8\bar{a}\bar{c}^3)\\+6\bar{a}\bar{b}\bar{c}^2(\bar{a}+\bar{b})(2\bar{b}\sin\gamma+\bar{a}\cos\gamma)\\+4\bar{a}\bar{b}\bar{c}^3(2\bar{a}+4\bar{b}+\bar{c})\sin\gamma\cos\gamma\\+\bar{c}^3(2\bar{a}^3+4\bar{a}^2\bar{b}-8\bar{a}\bar{b}^2+\bar{a}^2\bar{c}-16\bar{b}^3-4\bar{b}^2\bar{c})\cos^2\gamma\end{bmatrix}}{\bar{a}+2(\bar{b}+\alpha\bar{c})}\right\}$$

where $\alpha=1$, $\gamma=50^{\circ}$ (From 1996 and 2002 AISI Manuals) (17)

Warping constant for Hat sections without lips:

$$C_w=\frac{\bar{a}^2\bar{b}^2t}{12}\left\{\frac{2\bar{a}^3\bar{b}+3\bar{a}^2\bar{b}^2+\alpha\begin{bmatrix}48\bar{c}^4+112\bar{b}\bar{c}^3+8\bar{a}\bar{c}^3-48\bar{a}\bar{b}\bar{c}^2\\-12\bar{a}^2\bar{c}^2+12\bar{a}^2\bar{b}\bar{c}+6\bar{a}^3\bar{c}\end{bmatrix}}{6\bar{a}^2\bar{b}+(\bar{a}+\alpha 2\bar{c})^3}\right\}$$

where $\alpha=1$ (From this study and 1996, 2002 AISI Manuals) (18)

$$C_w=\frac{\bar{a}^2}{4}\left[I_x+(\bar{Y})^2A\left(1-\frac{\bar{a}^2A}{4I_y}\right)\right]+\left[\frac{2\bar{b}^2t\bar{c}^3}{3}-\bar{a}\bar{b}^2\bar{c}^2t+\frac{\bar{a}^2\bar{b}t\bar{c}^3\bar{Y}A}{3I_y}-\frac{4\bar{b}^2t^2\bar{c}^6}{9I_y}\right]$$

(From 1991 AISI Manual) (19)

Warping constant for equal leg angles without lips:

$C_w=0$

Warping constant for equal leg angles with lips:

$$C_w=\frac{\bar{a}^4\bar{c}^3t(4\bar{a}+3\bar{c})}{6[2\bar{a}^3-(\bar{a}-\bar{c})^3]}$$

(From this study and 1996, 2002 AISI Manuals) (20)

$$C_w=\frac{t^2\bar{a}^4\bar{c}^3}{18I_x}(4\bar{a}+3\bar{c})$$

(From 1991 AISI Manual) (21)

Warping constant for two channels with unstiffened flanges back-to-back welded:

$$C_w=\frac{t\bar{a}^2}{12}\left(\frac{8\bar{b}^3\bar{c}^3}{\bar{b}^3+\bar{c}^3}\right)$$

(This study and 1991, 1996, 2002 AISI Manuals) (22)

Warping constant for two channels with stiffened flanges back-to-back welded:

$$C_w=\frac{t\bar{b}^2}{3}(\bar{a}^2\bar{b}+3\bar{a}^2\bar{c}+6\bar{a}\bar{c}^2+4\bar{c}^3)$$

(This study, 1991, 1996, 2002 Manuals) (23)

where $\bar{a}$ is the length of web, $\bar{b}$ the length of flanges, $\bar{c}$ the length of unstiffened flange, t the thickness of plate element, and m is the distance between shear center and centerline of web as shown in Fig. 1.

Warping constants for all selective sections listed in the AISI Manuals were checked by both the derived and the AISI formulas. The differences were found to be negligible (See Tables 10~18).

7. Conclusions

The calculation of warping constant for a cold-formed steel open section is a tedious task for practicing engineers. This paper summarizes and discusses various integration formulas for evaluating C_w values of arbitrary cold-formed steel open sections. Based on this study, the following findings can be drawn.

1. This study presents the integration formulas for C_w in terms of more explicit numerical expressions for general open sections.

2. This study provides a detailed procedure for computing the C_w value of a general cold-formed steel open section. An illustrative example is provided to help practicing engineers calculate C_w values for sections not available in the AISI design manuals.

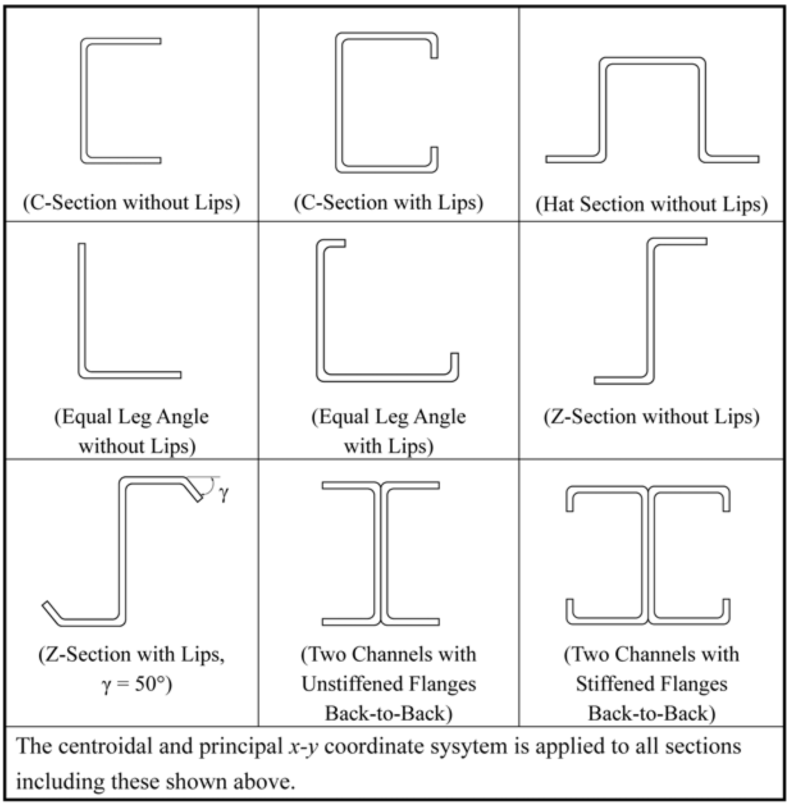

Figure 5. Nine practical sections provided in the AISI Manuals.

3. The C_w values calculated by the proposed procedure compare well with those tabulated in the current AISI design manuals. This means that the proposed procedure is accurate and reliable. The proposed computational procedure and formulas would be very useful for engineers when dealing with the C_w calculation for the sections not available in the design manuals.

4. Although sophisticated computer programs do exist, they may not be readily available for everyone. A simple numerical procedure utilizing Microsoft Excel was proposed to facilitate the computation of complex C_w.

5. Unlike the 1991 AISI formulas, the derived C_w formulas are a function of sectional dimensions only (ie $\bar{a}$, $\bar{b}$, and $\bar{c}$). During this study, discrepancies of C_w values tabulated in the 1991 AISI design manual were detected. For instances, C_w should be 0.108 in.^{6} (instead of 0.132 in.^{6}) for Section 4.0×1.125×0.060, and it should be 1.156 in.^{6} (instead of 1.56 in.^{6}) for Section 6.0×1.625 ×0.048.

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