Development of Human Body Model for the Dynamic Analysis of Footbridges under Pedestrian Induced Excitation

Sang-Hyo Kim¹, Kwang-Il Cho²*, Moon-Seock Choi³, and Ji-Young Lim⁴

¹Professor, School of Civil & Environmental Engineering, Yonsei University, A272 Gong-Hak Kwan A, Yonsei University, 262 Seongsan-no, Seodaemun-gu, Seoul, Korea
²Ph.D. Candidate, School of Civil & Environmental Engineering, Yonsei University, A365 Gong-Hak Kwan A, Yonsei University, 262 Seongsan-no, Seodaemun-gu, Seoul, Korea
³Doctoral course student, School of Civil & Environmental Engineering, Yonsei University, A365 Gong-Hak Kwan A, Yonsei University, 262 Seongsan-no, Seodaemun-gu, Seoul, Korea
⁴Master course student, School of Civil & Environmental Engineering, Yonsei University, A365 Gong-Hak Kwan A, Yonsei University, 262 Seongsan-no, Seodaemun-gu, Seoul, Korea

Abstract

Human-induced excitations are the most dominant source in vibration of footbridges. Such vibration effects are modeled as a mathematical time-domain force models in currently using softwares. However, such models are not capable of considering the dynamic effects of footbridges as they exclude the effects from precise human body model and human-structure interaction. Also, it is so complicated to analyze the dynamic effects from crowded walking load. In this study, a human body model for generating pedestrian excitation is developed which is adequate for considering the interaction between bridge and pedestrians on the footbridge. By using this model, the dynamic behaviors of footbridge are analyzed. It is also compared with the dynamic responses from the analysis using the time-domain force model and an experimental data. The dynamic responses from the analyses using the human body model are larger than the time-domain force model because of the human-structure interaction effects. Also, the proposed model shows good agreement with experimental result. Through a dynamic analysis for the footbridge under crowd-induced excitation, the developed numerical human body model is found to simulate the people walking in crowds effectively. From the analysis results, it is found that the partially grouped randomly walking stream could make even larger response than the synchronized crowded loads.

Keywords: footbridge, crowded pedestrian load, human body model, bridge-human interaction

1. Introduction

In recent decades there has been a trend towards to improve mechanical characteristics of materials used in footbridge construction. Engineers are now designing lighter, more slender and more aesthetic structures. As a result of these construction trends, many of these footbridges are becoming more susceptible to vibration when subjected to dynamic loads. In most cases the vibrations of footbridges lead to serviceability problems, i.e. the inconvenience of the pedestrians or in some extreme cases a bridge may no longer be used and has to be closed. In rare cases, safety problems may also arise due to over stressing and/or fatigue (Bachmann, 2002).

There have been many cases reported to have these problems, and they were especially emphasized to both public and professional researchers after the infamous swaying of the new and attractive Millennium Bridge in London during its opening day on June 10, 2000 (Dallard et al., 2001).

The human-induced dynamic loading occurs frequently and it is often regarded as dominant load for footbridges because it sometimes obstructs pedestrians to walk conveniently. There are two means of investigating footbridge vibration problem, namely experimental and numerical approaches. An experimental method requires considerable time and cost while the numerical approach represents an economical way to examine the dynamic behavior of a footbridge. To apply the appropriate force for the dynamic analysis of footbridges, various mathematical models of human-induced dynamic walking loads are proposed (Zivanovic et al., 2005), such as time and frequency domain force models. Although mathematical time and frequency domain force models are used in contemporary footbridge design, the dynamic analysis...
using these models is incapable of considering dynamic effects from the human-structure interaction. This is because these models are based on the data measured on a rigid surface. Also, it is so hard to apply time and frequency domain force model in analyzing dynamic responses from walking crowds. Therefore, a three-dimensional analysis program is developed in this study using the human body model. This program can effectively generate pedestrian excitation for the dynamic behavior of a footbridge under human-induced lateral and gravitational load.

2. Dynamic Analysis Methods for Footbridges under Human-induced Load

A time history dynamic analysis method using a commercial FE (finite element) analysis program with the application of the mathematical time-domain force model is the most common method to analyze the dynamic behavior of footbridges. However, this model is not suitable for dynamic analysis of footbridge since it cannot consider the effects from human-structure interaction that human makes to maintain his balance. To overcome this problem, a new human body model is proposed in this chapter. It is comparatively easier to consider these effects in the human body model. Also, it has advantages to apply complicated loading conditions such as loads from crowded pedestrians. In this chapter, the analysis methods using the mathematical time-domain model and the proposed mechanical human body model are briefly explained with their theoretical bases.

2.1. The dynamic analysis method using the mathematical time-domain force model

A time-domain force model is based on the assumption that both feet produce exactly the same periodic force. It is well-known that each periodic force $F_p(t)$ with a period $T$ can be represented by a Fourier series:

$$F_p(t) = G + \sum_{i=1}^{\infty} G\alpha_i \sin(2\pi f_i t - \phi_i)$$

where, $G$ is the person’s weight (N), $\alpha_i$ is the Fourier’s coefficient (dynamic load factors) of the $i$th harmonic, i.e., dynamic load factor (DLF), $f_i$ is the pacing frequency (Hz), $\phi_i$ is the phase angle of the $i$th harmonic, $i$ is the order number of the harmonic, and $n$ is the total number of contributing harmonics.

### Table 1. Correlation of pacing frequency, forward speed and stride length for walking (Bachmann and Ammann, 1987)

<table>
<thead>
<tr>
<th>Human motion</th>
<th>Pacing frequency (Hz)</th>
<th>Forward speed (m/s)</th>
<th>Stride length (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Vertical</td>
<td>Lateral</td>
<td></td>
</tr>
<tr>
<td>Slow</td>
<td>1.7</td>
<td>0.85</td>
<td>1.1</td>
</tr>
<tr>
<td>Normal</td>
<td>2.0</td>
<td>1.0</td>
<td>1.5</td>
</tr>
<tr>
<td>Fast</td>
<td>2.3</td>
<td>1.15</td>
<td>2.2</td>
</tr>
</tbody>
</table>

As shown in Eq. (1), the vertical force can be divided into static and dynamic components. The static component corresponds to a person’s weight and the dynamic component is the sum of harmonic functions with frequencies that are an integer multiple of the pacing frequency. The first to third harmonics are dominant; Eq. (1) can be simply rewritten in four terms as follows (Bachmann and Ammann, 1987; Bachmann et al., 1995):

$$F_p(t) = G + G\alpha_2 \sin(2\pi f_p t + \phi_2) + G\alpha_3 \sin(4\pi f_p t - \phi_3) + G\alpha_6 \sin(6\pi f_p t - \phi_6)$$

A pedestrian on a footbridge produces not only vertical force but also lateral force. Although the lateral force is relatively small compared to the vertical force, it is sufficient to produce strong vibrations in the case of laterally soft and hence low frequency structures. Fujino et al. (1993) noticed that the frequency of a person’s lateral head movement is 1 Hz which is twice of walking frequency. Lateral movement is ±10 mm when crossing a footbridge as shown in Fig. 1. As the frequency of lateral movement is half the vertical pacing frequency, the harmonic motion in lateral direction is half of that in vertical direction.

Table 1 shows the correlation of pacing frequency, forward speed and stride length according to human motion forms (Bachmann and Ammann, 1987). Normal walking is considered as the human motion for the mathematical time-domain force model in this study, hence the forward speed and the stride length is selected as 1.5 m/s and 0.75 m respectively. Bachmann and Ammann (1987) also proposed the dynamic load factors and the phase angles for normal walking as summarized in Table 2. By using the values on this table, the time-domain force models in vertical and lateral directions can be calculated using Eq. (2) as shown in Fig. 2. The resultant vertical and lateral time-domain force model is then applied to a FE model of a footbridge model.

![Figure 1. Lateral movement of person's head in walking](image-url)
2.2. The dynamic analysis method using the mechanical human body model

2.2.1. The mechanical human body model

Human walking motion is performed by complicated and elaborated movement of joints and muscles in a human body to maintain balanced attitude. Among the various components of foot-ground reaction force due to such complicated human walking motion, the vertical component is the most dominant component (Andriacchi et al., 1977). Therefore, a simple human body model (ISO, 1981) having vertical 2 DOF (degree of freedom) is considered as shown in Fig. 3. The total mass of this model is 736 N (75 kg).

The human body model has BCOM (Body Center Of Mass), the periodic vertical displacement of which characterizes normal human walking. The BCOM moves through a complete cycle of vertical motion with each step, or two cycles during each stride. To generate simulated pedestrian excitation for footbridges using a human body model, the vertical movement of the BCOM should be evaluated. Through the relationship between the BCOM acceleration, the vertical forces (Cavagna, 1975) and the forcing function due to a person's rhythmical body motion (Eq. (1)), Eq. (3-a) and Eq. (3-b) can be derived.

Table 2. Dynamic load factors and phase angles for normal walking (Bachmann and Ammann, 1987)

<table>
<thead>
<tr>
<th>Pacing rate (Hz)</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\phi_2$</th>
<th>$\alpha_3$</th>
<th>$\phi_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical</td>
<td>2.0</td>
<td>0.4</td>
<td>0.1</td>
<td>$\pi/2$</td>
<td>0.1</td>
</tr>
<tr>
<td>Lateral</td>
<td>2.4</td>
<td>0.5</td>
<td>0.1</td>
<td>$\pi/2$</td>
<td>0.1</td>
</tr>
</tbody>
</table>

![Figure 2. Time-domain force model in vertical and lateral direction](image-url)

(a) Vertical force

(b) Lateral force
acceleration of the BCOM. Although this equation has
same prototype with the time-domain force model, this
acceleration will produce pedestrian excitations in human
body model due to the vertical inertia. Also, some
additional excitations will be developed while considering
human-structure interaction force. Therefore, Bachmann’s
parameters such as $\alpha_i$ and $\phi_i$ are adopted to express an
external force. Since the human body model does not
have lateral DOF, the lateral human-structure interaction
is neglected and the lateral force described in Fig. 2(b) is
employed instead.

The equation of motion of the human body model is
derived from following generalized Lagrangian equation:

$$F_p(t) = mg + mg_i(t) = mg \left[ 1 + \sum_{i=1}^{n} \alpha_i \sin(2\pi f_i t - \phi_i) \right]$$  \hspace{1cm} (3-a)

$$a_z(t) = \sum_{i=1}^{n} \alpha_i \sin(2\pi f_i t - \phi_i)$$  \hspace{1cm} (3-b)

where $mg$ ($=G$) is the body weight, $a_z$ is the vertical
one of the direct integration methods. The equation of motion for the human body model is described in appendix.

Fig. 4(a), (b) shows time histories of ground reaction forces in terms of static and dynamic components. Red plot in Fig. 4(c) indicates the sum of static and dynamic components. As shown in Fig. 4(c), the human body model has more excitation compared to the time-domain force model which Bachmann suggested. This is because the human body model has considered the vertical inertia force, which is occurred from two lumped masses. These two models here are evaluated on the rigid surface; therefore, human-structure interaction effect is excluded.

It is necessary that these two models should have similar walking force since Bachmann's model represents real human loading on the rigid surface. In order to fit the walking force generated by the human body model to Bachmann's time-domain force model, dynamic load factors and phase angles are modified as listed in Table 3. Fig. 5 shows the time history of pedestrian excitation using the human body model, compared with the time-domain force model from which good agreement is found between the two models. The maximum and minimum values of the human body model after stabilization shows little difference of less than 1% compared to the time-domain force model as summarized in Table 4.

### Table 3. Modified dynamic load factors and phase angles for the human body model (vertical)

<table>
<thead>
<tr>
<th>Pacing rate (Hz)</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\phi_2$</th>
<th>$\alpha_3$</th>
<th>$\phi_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>0.28</td>
<td>0.05</td>
<td>0</td>
<td>0.18</td>
<td>0</td>
</tr>
</tbody>
</table>

### Table 4. The maximum and minimum walking force of the human body model and the time-domain force model after stabilization (Unit: N)

<table>
<thead>
<tr>
<th>Model</th>
<th>Human body model</th>
<th>Time-domain force model</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max.</td>
<td>1156.4</td>
<td>1156.7</td>
<td>0.02</td>
</tr>
<tr>
<td>Min.</td>
<td>460.1</td>
<td>459.1</td>
<td>0.21</td>
</tr>
</tbody>
</table>

### Figure 5. Walking force generated by human body model

### Figure 6. Flow chart of developed dynamic analysis program for the human body model

#### 2.2.2. Analysis algorithm considering human-structure interaction

The equations of motion for the bridge model and the human body model can be obtained in a matrix form. The two equations are coupled in terms of interacting force on a contact point. With the movement of a human body, the interacting force between both systems changes together with the contact points.

To analyze this interaction force, this study modified the program for dynamic behavior analysis developed in a previous study (Kim et al., 1999; Kim et al., 2005). This program includes matrix generation and decomposition, eigenvalue and eigenvector extraction, and a step-by-step transient response. The Newmark-$\beta$ method is used to integrate dynamic equations of the bridge under nonlinear human-induced load. Figure 6 shows a flow chart of the computer algorithm steps for the solution of the equations of human-structure interaction system.
3. Modeling of the Footbridge

To evaluate the proposed human body model, common footbridge is modeled using finite element method. It is a steel girder cable stayed footbridge located in a Seoul park in Korea (Fig. 7). The length and width of the footbridge is 99m (=4.5 m + 18 m × 5 + 4.5 m) and 3.3 m, respectively. Because this footbridge is designed to be slender and light weighted, it is required to determine the vibration characteristics. The FE model of this footbridge is designed with dimensional frame and truss elements as shown in Fig. 8. The damping ratio of the footbridge is assumed as 0.4% as suggested by Bachmann et al. (1995) for steel girder footbridges. In order to verify the bridge model, the natural frequencies and the deformation under constant pedestrian load (3.5 kN/m^2) are compared via commercial FE analysis program and the dynamic behavior analysis program developed in this study. The results of the natural frequencies and deformation are shown in Table 5 and 6. The natural frequencies and deformation values are similar from both programs.

Table 5. Comparison of natural frequencies of bridge models (Unit: Hz)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Commercial FEM analysis program</th>
<th>Developed dynamic analysis program</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>1.88</td>
<td>1.87</td>
<td>0.5</td>
</tr>
<tr>
<td>2nd</td>
<td>1.98</td>
<td>1.96</td>
<td>1.0</td>
</tr>
<tr>
<td>3rd</td>
<td>2.14</td>
<td>2.15</td>
<td>0.5</td>
</tr>
<tr>
<td>4th</td>
<td>2.21</td>
<td>2.16</td>
<td>2.3</td>
</tr>
<tr>
<td>5th</td>
<td>2.32</td>
<td>2.37</td>
<td>2.1</td>
</tr>
</tbody>
</table>

Table 6. Deformations at the midpoint on the first span under constant load (3.5 kN/m^2)

<table>
<thead>
<tr>
<th></th>
<th>Commercial FEM analysis program</th>
<th>Developed dynamic analysis program</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>16.5 mm</td>
<td>16.4 mm</td>
<td>0.6%</td>
<td></td>
</tr>
</tbody>
</table>
4. Dynamic Analysis of Footbridge under Pedestrian Excitation

Dynamic analyses are performed to investigate the dynamic behavior of the footbridge under pedestrian load. Dynamic behaviors of the footbridge using the developed human body model are compared with the results of the classical time-domain force model. The human body model is also verified with experimental measurements on the site. Based on the verified models, the effects of crowded moving pedestrian loads are studied.

4.1. Comparison of dynamic responses due to the time-domain force model and the human body model

To verify the validation of the human body model, the dynamic responses obtained by the human body model are compared with those from the time-domain force model. Table 7 shows the maximum dynamic responses due to a single walking pedestrian generated from the two models at the center of fourth span of the footbridge. Fig. 9 and 10 show the time histories of displacement and acceleration obtained by the two dynamic analysis methods.

Overall, the responses of the footbridge evaluated from the two methods show similar wave patterns. However, the maximum vertical displacement from the dynamic analysis using the human body model was 34% larger than that from the time-domain force model as tabulated in Table 7. Also, the maximum vertical acceleration using the human body model is nearly three times larger than the value from the time-domain force model. This is due to the dynamic effect and the human-structure interaction of the dynamic analysis method using the human body model. On the contrary, the difference of lateral response is very small, since there is no interaction between human and structure in lateral direction.

4.2. Comparison of analytical results with experimental results

When the new pedestrian model is proposed, it is usually verified by experimental data (Wu et al., 2008). In this chapter, human body model is verified via comparison study between experimental data and dynamic responses from the human body model. The test is conducted with under the walking load of five people (average mass: 72.5 kg). A 12-span cable stayed footbridge with 225 m length (4.5 m + 18 m @ 12 + 4.5 m), 3.3 m width was used. Specified properties of this bridge are same as chapter 3. Five pedestrians were making a group with 1 m distance walking through the two spans of the bridge, tried to synchronize their steps.

Two different walking features of the human body

| Table 7. Dynamic responses due to the time-domain force model and the human body model |
|-----------------|-----------------|-----------------|-----------------|
| Model           | Maximum displacement (mm) | Maximum acceleration (m/sec²) |
|                 | Vertical         | Lateral         | Vertical         | Lateral         |
| Time-domain force model | 0.632           | 0.053           | 0.074           | 0.011           |
| Human body model        | 0.847           | 0.060           | 0.260           | 0.044           |
| Difference           | 0.215           | 0.007           | 0.186           | 0.033           |

Figure 9. Time histories of displacement of the footbridge from dynamic analysis
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Model are used for comparison, namely, human body model R and human body model S. Human body model R applies different steps for every pedestrian, while human body model S is performed with same steps for all pedestrians, i.e., the steps are synchronized. Also, static analysis is compared to verify the model and the analysis result.

Table 8 and Fig. 11 show the responses of the footbridge obtained from the analysis and experiment. As shown in Table 8, the static displacement from analysis is similar to the experimental result; however, the analysis result is slightly overestimated. This indicates that the FE model properties are somewhat different from the properties of the footbridge. Also, the dynamic response from human body model S is close to the experimental result (difference: 4%). Due to the offset effects from

![Figure 10. Time histories of acceleration of the footbridge from dynamic analysis](image)

![Table 8. Maximum vertical displacements from analysis and experiment (unit: mm)](table)

<table>
<thead>
<tr>
<th>Static load</th>
<th>Human body model S*</th>
<th>Human body model R**</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analysis</td>
<td>1.20</td>
<td>1.71</td>
</tr>
<tr>
<td>Experiment</td>
<td>1.13</td>
<td>1.65</td>
</tr>
</tbody>
</table>

*: perfect synchronization, **: not synchronized

![Figure 11. Vertical displacement time histories from the analysis and experiment](image)
different steps, the response from human body model R is smaller than those from experiment and human body model S. Overall, it can be concluded that the dynamic analysis using the human body model is fairly acceptable, providing a reliable result.

4.3. Dynamic analyses for the footbridge under crowd-induced excitation

Through section 4.1 and 4.2, it can be confirmed that the human body model is suitable for the footbridge dynamic analysis. Thus, dynamic analysis is performed for the crowded pedestrians using human body model. Three different walking models are compared in this section, namely, the randomly walking human body model (human body model R), the synchronized model (human body model S) and the classical randomly walking model (classical random walking model $\sqrt{\lambda}$) which was proposed by the Matsumoto et al. (1978).

Specified descriptions regarding this model will be on the following paragraph. As described in section 4.2, all the pedestrians should have different steps to reflect the randomly walking people. Therefore, in human body model R, the longitudinal distances between the pedestrians and the individual steps are randomly defined. The loading features of human body model S and human body model R are shown in Fig. 12 and 13. Additionally, following statements are assumed during the analysis of human body model R. 1) A lanes that pedestrians are walking through are set to be three. (Distance between the lane is 1 m) 2) There are no distance limits on the bridge in the bridge-longitudinal direction 3) All the pedestrians are walking freely, i.e. unconstrained walking (Dallard et al., 2001).

Previous studies regarding to the load induced from the crowded pedestrians agreed that Matsumoto's model is fairly acceptable in randomly free walking pedestrian stream (S. Mouring, 1993, Grundmann et al., 1993). Matsumoto assumed that pedestrians arrived on the bridge following a Poisson distribution and they stochastically superimposed individual responses. He found that the response of the crowded pedestrians can be obtained by multiplying a single pedestrian response by the multiplication factor $\sqrt{\lambda T_0}$, where $\lambda$ is the mean arrival rate of pedestrians (pedestrians/second/width), and $T_0$ stands for the elapsing time to cross over the bridge. This multiplication factor can also be converted to $\sqrt{n}$, where $n$ means the number of pedestrians on the bridge at any time (S. Zivanovic et al., 2004). In the random vibration theory, if the responses due to $n$ equal and randomly distributed inputs are $\sqrt{n}$ times higher than the responses due to a single input, it means that inputs are absolutely uncorrelated (Newland, 1993). In the case of pedestrian load, it indicates that steps are absolutely unsynchronized (random). The relationship between the number of people and the multiplication factor is shown in Fig. 14.

In any case, Matsumoto's proposal was regarded as appropriate at least for footbridges with natural frequencies in the range of walking frequencies (1.8-2.2 Hz) (Zivanovic et al., 2004). However, Matsumoto's proposal was based on the assumption that the pedestrians are freely walking on the bridge. Grundmann et al. (1993) proposed that the upper limit for unconstrained free walking is 0.3 pedestrians/m$^2$. Instead of Matsumoto's proposal, he claimed other models of denser load cases, which are considering some synchronization between the neighbor pedestrians.

The dynamic behavior of the footbridge is estimated assuming four load cases of walking group; 99 pedestrians (density=0.33 pedestrians/m$^2$), 198 pedestrians (density=0.67 pedestrians/m$^2$), 297 pedestrians (density=1.0 pedestrians/m$^2$) and 445 pedestrians (density=1.5 pedestrians/m$^2$). As shown in Table 9, the case of 0.33

![Figure 12. Synchronized walking model (Human body model S)](image12)

![Figure 13. Randomly walking model (Human body model R)](image13)
pedestrians/m² represents the maximum density of free walking, and 0.67 pedestrians/m² for dense walking (Schlaich, 2002). 1.0 pedestrians/m² is assumed as very dense walking. This density is adopted because it is a recommended density to check the vibration serviceability (Seoul city, 2001). Finally, 1.5 pedestrians/m² represents extremely crowded density. These multiple pedestrian loads in each density cover whole bridge of span length 99m. For human body model R, the distances between the pedestrians and the walking steps are randomly decided. Therefore, the analysis should be repeated to find the representative values for each walking cases.

As indicated in Fig. 15, it can be concluded that overall six samples from human body model R are sufficient to get a reliable results, which guarantee the maximum 5% of coefficient of variation (COV). Thus, the responses of the human body model averaged from ten samples are appropriate enough. Fig. 16 exemplify the responses from human body model R for each pedestrian density. Because the responses from the human body model R are changing variously according to the each loading conditions, the most representative cases (the cases which are closed to the average value) are presented.

Fig. 17 shows the results of the dynamic responses from human body model R, human body model S and classical random walking model $\sqrt{n}$. Overall, vertical responses are 3~4 times bigger than lateral responses. Moreover, increase of human density induces larger displacement and acceleration. For the density of 1 pedestrians/m², the vertical displacement response from human body model S is 27% larger than that of human body model R. For the vertical acceleration, human body model S shows 49% larger response than human body model R. In the density of 0.33 pedestrians/m², human body model R induces acceleration which is 28% larger than response from human body model S.

Since the classical random walking model $\sqrt{n}$ is based on the assumption that pedestrians are walking freely, in fact, it is restricted to be applied to the cases with density larger than 0.30 pedestrians/m². However, it is applied to compare human body model R, as it is assumed as free walking, because there is no previous study that analyzed crowded human load with step-by-step moving model on

**Table 9. Density and pedestrian numbers on each traffic type**

<table>
<thead>
<tr>
<th>Traffic type</th>
<th>Density</th>
<th>Number of pedestrians</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free</td>
<td>0.33 pedestrians/m²</td>
<td>99 men</td>
</tr>
<tr>
<td>Dense</td>
<td>0.66 pedestrians/m²</td>
<td>198 men</td>
</tr>
<tr>
<td>Very dense</td>
<td>1 pedestrians/m²</td>
<td>297 men</td>
</tr>
<tr>
<td>Extremely crowded</td>
<td>1.5 pedestrians/m²</td>
<td>445 men</td>
</tr>
</tbody>
</table>

**Figure 14.** Relationship between number of persons and multiplication factor (Matsumoto et al., 1987)

**Figure 15.** COVs for various number of samples (vertical)
footbridges. As shown in Fig. 17(a) and (c), the responses from human body model R are smaller than the responses from human body model S and larger than that of classical random walking model. It can be concluded that the human-structure interaction and the excitations that human makes to maintain his balance have somehow affected the responses to have a larger value than classical random walking model.

For the vertical acceleration shown in Fig. 17(c), the responses from human body model R are larger than the values from human body model S. This is because human models configured in human body model S have same distances between each other (Fig. 12). On the contrary, human body model R has possibilities to include much shorter distances between the pedestrians (Fig. 13). In human body model R, the concentration of load could induce larger responses compared to human body model S. Therefore, it can be concluded that the stream of partially grouped pedestrians sometimes produce larger responses than synchronized walking load. This effect is also shown in the result of lateral acceleration (Fig. 17(d)). However, since lateral responses are much smaller than vertical responses, the effect in lateral responses does not require serious consideration.

Such results indicate that further study should be performed to develop more suitable multiplication factors.
that can consider these additional effects. Also, study on lateral and vertical synchronization effects using the human body model should be performed to improve the result from crowded pedestrian loads.

5. Conclusions and Remarks

In this study, dynamic analyses of footbridges under complicated walking load are carried out considering human-structure interaction using a human body model, for the generation of pedestrian load. For the verification of the dynamic analysis method using the human body model, it is compared with other analysis methods and experimental data. Additionally, dynamic analyses of a footbridge under crowd-induced excitations are conducted. The following conclusions are drawn from the results.

A human body model is developed, which can generate pedestrian excitations using excited BCOM (Body center of mass) model. This model can be applied systematically to model the complicated pedestrian loading patterns, including the synchronized or random movement of huge pedestrian group.

The dynamic analysis using the time-domain force model may underestimate the responses, if compared to those from the human body model, since the human-structure interaction is excluded.

In the case of the footbridge used in this study, it is found that the synchronized walking produces generally larger dynamic responses than those from the random walking up to 43%. However, when the pedestrians are forming a low density walking stream, synchronized walking and random walking show similar results.

It is found that the randomly walking crowded loads are not accurately represented by multiplying Matsuno’s multiplication factor to a single person response. Further study should be performed to investigate the simple and engineer accessible analysis procedure for various types of footbridge.
Acknowledgment

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References


Appendix: Equations of motion of human body model

1. Relative displacement

   Upper spring:
   \[ R_{s1} - z_{s1} - z_{a1} \]  \hspace{1cm} (A.1)

   Lower spring:
   \[ R_{s1} - z_{a1} - z_{b1} \]  \hspace{1cm} (A.2)

2. Energy

   Kinetic energy:
   \[ T = \frac{1}{2} m_1 z_1^2 + \frac{1}{2} m_2 z_2^2 \]  \hspace{1cm} (A.3)

   Potential energy:
   \[ V = \frac{1}{2} R_{s1} K_{s1} + \frac{1}{2} R_{s2} K_{s2} \]  \hspace{1cm} (A.4)

   Dissipation energy:
   \[ D = \frac{1}{2} R_{s1}^2 C_{s1} + \frac{1}{2} R_{s2}^2 C_{s2} \]  \hspace{1cm} (A.5)

3. Equations of motion

   \[ m_1 \ddot{z}_{a1} + F_{s1} + F_{c1} = -m_1 g \]  \hspace{1cm} (A.6)

   \[ m_2 \ddot{z}_{b1} - F_{s1} + F_{c1} = -m_1 g \]  \hspace{1cm} (A.7)

where,

   \[ F_{s1} - K_{s1} R_{s1} + F_1 \]  \hspace{1cm} (A.8)

   \[ F_{c1} - C_{s1} R_{s1} \]  \hspace{1cm} (A.9)

   \[ F_{s1} - C_{s1} R_{s1} \]  \hspace{1cm} (A.10)

   \[ F_{c1} - C_{s1} R_{s1} \]  \hspace{1cm} (A.11)

   \[ z_{s1}, z_{a1}, z_{b1} \]: displacement of upper and lower mass and foot

   \[ m_{s1}, m_{a1}, m_{b1} \]: upper and lower mass

   \[ K_{s1}, K_{s2} \]: spring constant of upper and lower spring

   \[ C_{s1}, C_{s2} \]: damping coefficient of upper and lower dashpot