Analysis of the Shear Lag Effect in Composite Bridges with Complex Static Schemes by means of a Deck Finite Element

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Abstract

This paper presents the derivation of a finite element formulation for the analysis of composite decks accounting for partial interaction theory and shear-lag effects. The particularity of the proposed element, referred to as the deck finite element, relies on its ability to capture the structural response of complex static schemes, such as those specified for arch, bow-string and cable stayed bridges, while preserving the ease of use of a typical line element. For these particular bridge solutions, stress concentrations may be induced in the slab by the application of concentrated forces, i.e. due to the anchorage of prestressing cables or stays, or due to the presence of web members in arch bridges. The ease of use of the proposed deck finite element is outlined considering two case studies for which the calculated results have been compared against those obtained using a more refined model implemented using shell elements in a commercial finite element software.

Key words: Bridges, Steel-concrete composite decks, shear-lag, finite element method, effective width

1. Introduction

Composite steel-concrete beams have been extensively used over the last decades for bridge applications. In their simplest typology longitudinal steel members have been specified to act compositely with the supported reinforced or prestressed concrete slabs. This solution has been commonly used for viaducts or flyovers with medium span lengths in the range of 40-100 m with frequent implementations using only two longitudinal steel joists even with wide slabs, usually referred to as twin-girder decks. For these slab-girder systems it has already been shown in the literature that the usual assumption of bending theory, according to which the plane cross sections remain plane after loading, is not realistic and non uniform stress distributions usually arise in the slab reducing its effective width, i.e. shear-lag effects. (Von Karman, 1924; Reissner, 1946; Dezi and Mentrasti, 1985; Gjelsvik, 1991) In addition to the simple scheme of multi-span continuous beams, composite decks have also been adopted in combination with other structural elements in complex static schemes such as those used in arch bridges, bow-string bridges and cable stayed bridges. For these particular solutions stress concentrations may also be induced in the slab by the application of concentrated forces, i.e. due to the anchorage of prestressing cables or stays, or due to the presence of web members in arch bridges. For these particular static schemes, the analytical and numerical models proposed for normal slab-girder solutions (e.g., Dezi et al., 2006) as well as the method of the effective width proposed by the main codes of practice (e.g., prEN 1994-2 2003) are inadequate to capture the complex stress distributions in the slab which require the use of refined finite element analyses by modelling the bridge systems with shell or solid finite elements. From a designer’s viewpoint, the use of these complex numerical finite element models provide very detailed information on the response of the structure even if at the expense of an extensive post-processing effort to transform the numerical output information in meaningful variables, i.e. stress resultants, useful for design purposes.

In this context, the purpose of this paper is to propose a finite element, hereafter referred to as a deck finite element, capable of capturing the structural response of complex static schemes while preserving the ease of use of typical line elements. In fact, the particularity of the proposed formulation relies on its ability not only to depict the warping behaviour due to the shear-lag effects originated due to vertical loading or prestressing actions, as would occur in normal slab-girder systems (Gara et al., 2008; Macorini et al., 2006; Sun and Bursi, 2005; Dezi et
al., 2001; Prokic, 2001; Sedlacek and Bild, 1993), but to account for the concentrated forces, and their produced stress distribution, which occur in complex three-dimensional bridge geometries.

The initial part of the paper presents the analytical model required to describe the partial interaction behaviour of a composite steel-concrete deck with shear-lag effects followed by the derivation of a finite element formulation and its implementation in a three-dimensional spatial domain. Finally, two numerical applications are presented to outline the ability of the proposed deck finite element to capture the complex stress distributions of cable-stayed and arch bridges. These results have also been compared against those calculated using a shell element model implemented in the commercial finite element software SAP2000 (SAP2000, 2004).

The proposed work intends to complement the analysis tools available to bridge designers. For example, an analysis carried out using the proposed line element would be able to capture the distribution of internal actions accounting for the effects of the slab warping. Its main advantages rely on the fact that the output of the analysis would consist of both the design internal actions (whose calculations already consider the effects of shear-lag) and of the effective width to be used for the verification of the cross-sectional capacity. It is worth highlighting that line elements based on partial interaction theory available in the literature, but without the ability to account for shear-lag effects, (Gara et al., 2006; Ranzi et al., 2006) could also be used for this purpose specifying values for the slab effective widths as an input data before the analysis. While this is a possibility for simple composite bridges, e.g. twin-girder decks, for which guidelines for the determination of the effective widths are available in design codes, this is not feasible for bridge decks part of more complex static schemes as no guidance is available for their slab effective width values. In this context, the proposed line element represents an attractive option as it does not suffer from this drawback. At the same time, these advantages are counterbalanced by its inability to capture the response of unbalanced loads placed across the width of the bridge. The authors do not feel that this limitation compromises the usefulness of the proposed element which could be certainly used in preliminary design and costing, and as an efficient tool to determine effective width values for complex bridge systems.

2. Analytical model

2.1. Model assumptions

A prismatic composite beam formed by two layers, as shown in Fig. 1, is considered. In its undeformed state, the composite beam occupies the cylindrical region \( V = A \times [0, L] \) generated by translating its cross-section \( A \), with boundary \( \partial A \), along a rectilinear axis which is orthogonal to the cross-section and is assumed to be parallel to the \( x_1 \) axis of the ortho-normal member reference system \( \{O; x_1, x_2, x_3\} \). For generality, the formulation is derived for a beam segment of length \( L \) and about an arbitrary coordinate system (Fig. 1).

The composite cross-section is represented as \( A = A_1 \cup A_2 \), where \( A_1 \) and \( A_2 \) are the cross-sections of the top and bottom elements, which, for ease of reference, are referred to as elements 1 and 2 respectively, and it is assumed to be symmetric about the plane of bending with the coordinate plane \( x_2 x_3 \) being taken as the plane of symmetry.

Without any loss of generality, the composite cross-section here represented is that of a typical composite steel-concrete beam in which \( A_1 \) consists of the reinforced slab and it is further subdivided into \( A_1 \) and \( A_c \), which represent the concrete component and the reinforcement respectively, i.e. \( A_1 = A_1 \cup A_c \), while \( A_2 \) represents the cross-section of the steel joist only and it is denoted as \( A_c \).

The composite action is provided by a connection which is assumed to be uniformly spread along a rectilinear line \( \Lambda \) at the interface between the two layers, whose domains consist of \( \{x_2, x_3\} = \{x_2^{SC}, x_3^{SC}\} \), where \( x_2^{SC} \) and \( x_3^{SC} \) are defined in Fig. 1.

2.2. Displacement and strain fields

The displacement field adopted in the formulation considered consists of the axial displacement of the steel member at the level of the reference axis \( u_1(x_1) \), the vertical displacement \( u_2(x_1) \), the relative longitudinal slip between the two layers \( \Gamma(x_1) \) and the intensity shear-lag function related to the cross-sectional warping \( \omega(x_1) \) respectively (Reissner, 1946). For ease of notation these are simply referred to as \( u_1, u_2, \Gamma \) and \( \omega \) in the following.

The expressions for the rotation and the curvature along the beam can be obtained by differentiating the expression for the deflection with respect to the coordinate along the member \( x_1 \) and these are denoted as

![Figure 1. Typical composite deck beam and cross-section.](image-url)
$u_x^2$ and $u_y^2$ are defined as the derivative with respect to $x_1$. The displacement field $\mathbf{d} = [d_1, d_2, d_3]_T$ adopted in the proposed model is defined as (Fig. 2)

$$d_1(x_1, \xi, \eta) = \begin{cases} u_1(x_1) = \bar{u}_1(x_1) \psi_1(x_1) + \bar{u}_2(x_1) \psi_2(x_1) \gamma_1(x_1) & \forall (x_1, \xi, \eta) \in A_1 \\ u_2(x_1) = -\bar{u}_2(x_1) \psi_2(x_1) + \bar{u}_3(x_1) \psi_3(x_1) \gamma_1(x_1) & \forall (x_1, \xi, \eta) \in A_2 \\ d_3(x_1, \xi, \eta) = 0 & \forall (x_1, \xi, \eta) \in A_2 \end{cases}$$

where $\psi_1$, $\psi_2$, and $\psi_3$ are the generic warping functions defined in the next section. The non-zero components of the strain tensor, based on the assumed displacement field, are

$$\epsilon_{11}(x_1, \xi, \eta) = \frac{\partial u_1}{\partial x_1}$$

$$\epsilon_{22}(x_1, \xi, \eta) = \frac{\partial u_2}{\partial x_1}$$

$$\epsilon_{33}(x_1, \xi, \eta) = \frac{\partial u_3}{\partial x_1}$$

3. Weak Formulation

The global equilibrium of the structural problem can be obtained by enforcing the virtual work theorem. Collecting the generalized displacements in the vector $\delta \mathbf{u} = [u_1, u_2, \Gamma, \omega]_T$, its weak formulation can be expressed in the following concise form for any kinematically admissible variations of the displacement $\delta \mathbf{u}$ as

$$\int_{\Omega} \mathbf{r} : \mathbf{D} \delta \mathbf{u} - \int_{\Omega} (\mathbf{p} \cdot \mathbf{H} \delta \mathbf{u}) dV$$

in which $\mathbf{r}$ represents the relevant stress resultants, $\mathbf{p}$ collects the external loading caused by body forces $\mathbf{b} = [b_1, b_2, b_3]_T$ and surface forces $\mathbf{q} = [q_1, q_2, q_3]_T$ expressed as

$$\mathbf{p} = \begin{bmatrix} q_1 d_1 + b_1 d_1 \\ q_1 d_1 + b_1 d_2 \\ q_2 d_1 + b_2 d_1 \\ q_2 d_1 + b_2 d_2 \\ q_2 d_1 + b_2 d_3 \\ q_2 d_1 + b_2 d_4 \\ q_2 d_1 + b_2 d_5 \end{bmatrix}$$

$$\mathbf{q} = \begin{bmatrix} q_1 x_1 d_2 - b_1 x_2 d_2 \\ q_1 x_1 d_2 - b_1 x_2 d_3 \\ q_1 x_1 d_2 - b_1 x_2 d_4 \\ q_1 x_1 d_2 - b_1 x_2 d_5 \end{bmatrix}$$

2.3. Warping function

The expressions for the warping function $\psi(\tilde{x})$ introduced in the proposed model are derived from a simplified problem where the slab is considered to be a rectangular membrane subjected to two longitudinal uniformly distributed loads, acting along the lines of the steel beam webs, and to membrane uniformly distributed longitudinal normal stresses applied at the end cross sections so as to fulfill global equilibrium. (Dezi et al., 2001) Due to the small slab thickness, it is assumed to be constant on the slab depth and, in order to catch the planarity loss of the whole slab taking account of the actual beam spacing, the adopted $\psi(\tilde{x})$ consist of the three parabolic branches outlined in Fig. 2(b) and defined as

$$\psi(\tilde{x}) = \begin{cases} \frac{\tilde{x}}{B} & -B \leq \tilde{x} \leq -B \\ \frac{\tilde{x}}{B} - \left( \frac{B}{B} \right) & -B < \tilde{x} \leq +B \\ \frac{\tilde{x}}{B} - \left( \frac{B}{B} \right) & +B < \tilde{x} \leq +B \end{cases}$$

Figure 2. Displacement field.
while $D$ and $H$ are the formal linear differential operators defined as

$$
D = \begin{bmatrix}
0 & 0 & 0 \\
0 & -\partial & 0 \\
\partial^2 & 0 & 0 \\
0 & 0 & 1 \\
0 & 1 & 0
\end{bmatrix} \quad H = \begin{bmatrix}
1 & 0 & 0 \\
0 & 10 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
$$

(6a,b)

where $\partial = \partial / \partial z$.

For the purpose of this paper, it is assumed that the concrete, the reinforcing bars and the steel joist behave in a linear-elastic fashion in both compression and tension, while the cross-sectional properties related to the warping function are defined as

$$
K = \begin{bmatrix}
E_c A_c & 0 & -E_c B_s & 0 & 0 & 0 \\
0 & E_s A_s + E_r A_r & -E_c B_s & 0 & 0 & 0 \\
-E_c B_s & -E_c B_s & E_s B_w + E_r B_w & E_s B_w & 0 & 0 \\
E_s I_x + E_r I_y & E_s I_y & -E_s I_x & E_s I_y & -E_s I_x & -E_s I_y \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

(7)

and $A_c$, $A_s$, $A_r$, $B_s$, $B_r$, $I_x$, $I_y$, and $I_z$ are the area, the first and second moments of area of the concrete component, the reinforcement and the steel joist respectively, while the cross-sectional properties related to the warping function are defined as

$$
B_{qc} = \int x d\alpha; \quad I_{2qc} = \int x^2 d\alpha \\
I_{qc} = \int y d\alpha; \quad J_{qc} = \int y^2 d\alpha \\
B_{qc} = \int y d\alpha; \quad I_{2qc} = \int y^2 d\alpha
$$

(9a,b,c,d)

4. Finite Element Formulation

4.1. Deck finite element in local coordinates

A numerical solution of the problem can be obtained using the finite element method by starting from the weak formulation. This is achieved by introducing a linear combination of interpolating shape functions for the unknown displacements in the global balance condition and $E_C$, $E_r$, $E_s$ are the relevant Young moduli for the concrete, steel reinforcement and steel joist respectively. Similar linear-elastic assumptions are adopted for the shear connection introducing a stiffness modulus $p$ that relates the shear flow force per unit length $q$ to the slip at the interface $\Gamma$.

Based on the adopted constitutive models, the internal stress resultants can be expressed in compact form as

$$
r = KDs
$$

in which

$$
\mathbf{s} \equiv \mathbf{N}_e \mathbf{s}_e
$$

(10)

where $\mathbf{s}_e$ is the vector of the unknown nodal displacements and $\mathbf{N}_e$ is the interpolation matrix defining the shape functions.

A local reference system is introduced for the proposed deck finite element (Fig. 3) with the origin located at node $i$. If $\bar{x}_1$ is the longitudinal axis of the element, oriented from joint $i$ to joint $j$, axes $\bar{x}_2$ and $\bar{x}_3$ complete an orthonormal reference system. The element nodal displacements are defined according to the positive directions of the local reference axes. For each node, three displacement components and three rotation components are considered. Two additional degrees of freedom are introduced to define the beam-slab interface slip $\Gamma$ and the shear-lag function $\phi$ that measures the intensity of the slab warping. These last two generalized displacements are defined as scalar quantities and thus do not depend on the adopted reference system.

Substituting equation (10) into the weak formulation expressed by equation (4) yields

![Figure 3. Deck finite element.](image)
which can be re-arranged as

\[ \mathbf{f}_e = -\mathbf{K}_e \mathbf{s}_e \]

where \( \mathbf{K}_e \) is the stiffness matrix of the deck finite element, while \( \mathbf{f}_e \) and \( \mathbf{s}_e \) represent the generalized nodal displacements and forces of element \( e \) as

\[
\begin{bmatrix}
\mathbf{s}_e^T \\
\mathbf{f}_e^T
\end{bmatrix} = \begin{bmatrix}
\mathbf{s}_i \\
\mathbf{f}_i
\end{bmatrix} \quad \text{and} \quad \begin{bmatrix}
\mathbf{s}_e^T \\
\mathbf{f}_e^T
\end{bmatrix} = \begin{bmatrix}
\mathbf{s}_i \\
\mathbf{f}_i
\end{bmatrix}
\]

\[ (13a,b) \]

\[
\mathbf{s}_a = [u_\alpha, \Gamma_\alpha, \omega_\alpha, q_\alpha, \phi_{1\alpha}, \phi_{2\alpha}, \phi_{3\alpha}, \Gamma_\alpha, \omega_\alpha] \quad \text{and} \quad f_a = [f_{1\alpha}, f_{2\alpha}, f_{3\alpha}, m_{1\alpha}, m_{2\alpha}, m_{3\alpha}, q_\alpha, \beta_\alpha]
\]

\[ (14a,b) \]

For the purpose of this study, the vertical displacements are approximated by third order polynomial functions whereas the longitudinal displacement of the steel beam, the interface slip and the shear-lag function are approximated by second order ones. The use of the internal node, related to the approximated generalised displacements \( u_\alpha, \Gamma \) and \( \omega \), is necessary to guarantee a consistent interpolation field to avoid locking problems. By means of a static condensation, the problem can be expressed only in terms of the ten displacements of the external nodes (Fig. 3).

### 4.2 Deck finite element in global coordinates

The deck element can be oriented freely in the three dimensional space in which an ortho-normal global reference system is introduced. If \( a_\alpha (\alpha = 1, \ldots, 3) \) are the unit vectors of the global system and \( \bar{a}_\alpha (\alpha = 1, \ldots, 3) \) are those of the local reference axes (Fig. 4), the rotation operator \( \mathbf{R}_e \) is defined as

\[
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & I_{22} & 0 \\
0 & 0 & 0 & I_{22}
\end{bmatrix}
\]

\[ (15) \]

where

\[
\begin{bmatrix}
\bar{a}_1 : a_1 \\
\bar{a}_2 : a_2 \\
\bar{a}_3 : a_3 \\
\bar{a}_4 : a_4
\end{bmatrix} ; \quad I_{22} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}
\]

\[ (16a,b) \]

Standard finite element procedures enable the following transformations from local to global coordinates for the element stiffness matrix, the nodal displacement vector and the nodal force vector

\[
\mathbf{K}_e = \mathbf{R}_e^T \mathbf{K}_e \mathbf{R}_e; \quad \mathbf{s}_e = \mathbf{R}_e \mathbf{s}_e; \quad \mathbf{f}_e = \mathbf{R}_e \mathbf{f}_e
\]

Equations (17) result in the rotation of the vectorial entities of the three-dimensional space (displacements and rotations) whereas the scalar degrees of freedom (interface slip and shear-lag function) and the relevant forces (longitudinal shear flow and bi-moment) are not modified.

### 4.3 Element connectivity

The proposed deck element can be connected to other kinds of finite elements, i.e. beam and truss elements, using standard finite element assembling procedures. It is worth highlighting that the generalised displacement field adopted in the proposed formulation enables to connect only the steel member of the deck finite element to surrounding elements, as would be often the case in practice where connections between steel members are privileged. Certainly, other sets of generalised displacements could be easily specified if necessary.

Due to the complexity of some bridge static schemes, i.e. cable-stayed bridges, rigid links have been also defined for the proposed deck element to account for eccentric connections between members as illustrated in Fig. 4(b).

In the assembling procedure for a model comprising different finite elements, i.e. deck, beam and truss elements, the admissible degrees of freedom associated to each joint are inherited from the element with the higher level of hierarchy among those connected to the joint. The stiffness matrix assembly is thus simple and descends from the element connectivity on the basis of the structural topology by enforcing the consistency of the nodal displacements and can be performed as usual by a summation procedure suitably selecting the contributions due to the various elements (Smith and Griffiths, 1998).

### 4.4 Restraints and constraints

The particularity of the proposed element requires two additional restraints to be specified, related to the interface.
slip and the slab warping, in addition to the usual external restraints corresponding to the six displacement components. In fact, the slip restraint can be used to model the deck with rigid shear connections whereas the restraint on the slab warping can simulate the presence of rigid structural elements, e.g. end transverse beams. In this sense, the case of flexible transverse beams limiting the slab warping can be caught by defining special generalized springs.

In the cases where members connected to and surrounding the deck finite element do not provide sufficient restraint to the deck rigid body motion ill-conditioned situations may arise. This is a consequence of the fact that, in its current condition zero stiffness is associated to the torsion of the deck and to the bending in the plane \( \vec{x}_1\vec{x}_3 \). This is inhibited in the proposed formulation by the introduction of a constraint enforcing rigid motions of the deck under torsion and under bending on plane \( \vec{x}_1\vec{x}_3 \), therefore leading to a partially rigid deck. If \( O \) is the master joint of the deck, and \( P \) is the generic slave joint, the necessary constraints are expressed as (Fig. 4)

\[
(\vec{a}_3 \otimes \vec{a}_1) \alpha^T P - (\vec{a}_3 \otimes \vec{a}_1) \alpha^T P_O - (P_O - P) \times (1 - \vec{a}_3 \otimes \vec{a}_1) \alpha^T P_O = 0 \quad (18a)
\]

\[
(1 - \vec{a}_3 \otimes \vec{a}_1) \alpha^T P - (1 - \vec{a}_3 \otimes \vec{a}_1) \alpha^T P_O = 0 \quad (18b)
\]

where \( \vec{a}_3 \otimes \vec{a}_1 \) and \( 1 - \vec{a}_3 \otimes \vec{a}_1 \) are linear operators which project vectors of the three dimensional space along \( \vec{a}_3 \) and on planes orthogonal to \( \vec{a}_3 \), respectively, and \( \alpha^T P - [U_1, U_2, U_3]^T \) and \( \alpha^T [\Phi_1, \Phi_2, \Phi_3]^T \) are the vectors which contains the translation and rotation components of nodal displacements with respect to the global reference system, respectively. Denoting \( A \) the constraint operator obtained by suitably arranging the conditions outlined in equations (18), the displacements \( s \) of the system can be expressed as \( s - A \tilde{s} \), where \( \tilde{s} \) are the residual displacements. The global problem is thus condensed into

\[
A^T K A \tilde{s} - A^T f = 0
\]

which is well-conditioned.

5. Applications

Two typologies of bridges, i.e. one cable-stayed bridge and one arch bridge, are considered in this section to outline the ease of use of the proposed deck finite element and how this can be easily combined with other truss and frame elements to model complex bridge systems. The results obtained with the deck finite element are compared in the following against those calculated with a model implemented using shell elements in the commercial FE software SAP2000 (SAP2000, 2004).

5.1. Cable-stayed bridge

The case study considered for the typology of cable-stayed bridge is shown in Fig. 5 and follows the general layout and dimensions of the Quincy Bridge (US). The two finite element models implemented for this application are shown in Fig. 6. In particular, Fig. 6(a) depicts the model prepared using the proposed deck finite element combined with the common truss and frame elements, while Fig. 6(b) outlines the meshing adopted in SAP2000 where both steel beams and slab have been modelled using shell elements. For the former model, rigid links have been specified at the ends of the deck element to ensure the correct positioning of the stays at the sides of the bridge.

The loading conditions adopted in the analyses are illustrated in Fig. 7(a) which include both a symmetric component simulating the deck self-weight and a non-symmetric one representative of a possible traffic load applied over half of the internal span. At the beginning of the simulations the bridge stays are pretensioned to

Figure 5. General layout and dimensions of the Quincy Bridge (US).
Figure 6. Finite element modeling of the Quincy Bridge.

(a) FE model using the proposed deck finite element  
(b) Shell FE model in SAP2000

Figure 7. Cable-stayed bridge.
provide an initial condition of zero deflection throughout the length of the bridge when subjected to its self-weight only. The additional deflection of the bridge due to the non-symmetric loading is depicted in Fig. 7(b) which illustrates how both the proposed deck finite element and the shell FE models produce equivalent results. Very good agreement is also shown for the values for the longitudinal stresses calculated in both the slab and the steel joints. Figure 7(c) plots the stresses calculated at mid-height of the slab along the bridge length at the location of the steel member axis and along the centre-line of the deck while the stresses obtained for the top and bottom steel flanges are illustrated in Fig. 7(d). The variation of the effective width is determined based on the calculated stress distributions and is illustrated in Fig. 7(e). It is worth highlighting that the localised minimum values observed for $B_{ref}$ near the end supports and near mid-span are not relevant from a design viewpoint as related to bridge segments along which the stress levels are relatively low. This is also shown in Fig. 8 which outlines the stress distributions across the deck width at mid-height of the concrete depth at four different locations along the bridge defined in Fig. 7(a). Also in this case the results calculated using the proposed deck element well agree with those obtained using the more refined shell FE model. Similar considerations can be drawn for the concrete stresses calculated at the top and bottom of the slab and, for this reason, their graphical representation has been omitted. It is worth highlighting how the maximum stresses in the concrete occur at both

![Figure 8. Transverse distribution of the normal stresses of the concrete slab at different locations along the bridge length.](image)

![Figure 9. Deflected shape of the cable-stayed bridge obtained using the proposed Deck FE model.](image)

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the centre-line of the deck and along the member axis of the steel beams depending on the location of the cross-section considered. In fact, for sections 1 and 3 defined in Fig. 7(a), the peak values for the concrete stresses take place at the location of the steel joist (Fig. 8) while for sections 2 and 4 these are observed in the middle of the deck width.

Comparisons have also been carried out for the axial forces induced in the bridge stays using the two finite element models (Fig. 6) and the maximum observed error is very small, i.e. less than 0.4% as specified in Table 1. An overview of the deflected shape of the whole model is illustrated in Fig. 9.

5.2 Arch bridge

The layout and geometry of the bridge utilised for the second case study are inspired to those of the arch bridge at Antrenas in France (Virlogeux et al., 1994) which are presented in Fig. 10. As in the case of the cable-stayed bridge, two numerical models have been implemented. Figure 11(a) outlines the one based on the use of the proposed deck finite element together with the common truss and frame elements. In this case the two triangular concrete chords (Fig. 10) are included in the deck as two longitudinal beams rigidly connected to the slab by means of suitable external restraints to the interface slip. To enable the correct geometry and connectivity of the underlying truss system to be specified, the deck elements are modelled along the centre-line of the bridge deck and are connected to the truss elements by means of rigid links. The more refined model in which the slab has been represented by means of shell elements has been prepared using SAP2000 as outlined in Fig. 11(b).

Two distributed loads, i.e. a symmetric one and a non-symmetric one representative of the deck self-weight and of the traffic load respectively, have been considered as illustrated in Fig. 12(a).

Figure 12(b) depicts the variation of the deck deflection calculated using the two finite element models which appear to be in good agreement. A wide range of concrete stresses are reported in Figs. 12(c-e) to outline the complex distribution along the bridge. For completeness, these values have also been plotted at different levels of the concrete depth, i.e. at the top, mid-height and bottom of the slab, to better describe the observed stress variations. Based on Figs. 12(c-e) it can be noted that while both numerical models provide similar results for the concrete stresses calculated at mid-height of the slab, some discrepancies are observed for the peak values induced in the top and bottom faces of the slab when calculated at the locations where the underlying truss members connect to the upper deck. This is caused by the inability of the deck element to capture the local flexural deformations at these connections.

Figure 13 shows the transverse distribution of stresses calculated at four selected cross sections specified in Fig. 12(a) from which it can be observed that the proposed deck element performs well when determining stresses across the bridge width at mid-height of the slab. Maximum stresses are observed to occur along the centre-line of the bridge for cross-sections 1, 3 and 4, which are located away from connections with the underlying truss. On the other hand, peak values at cross-section 2 are shown to occur near the first web member connection to the deck.

To provide a better overview of the performance of the numerical model formed by the deck, truss and frame elements the axial forces calculated for each truss member have been compared against those obtained using the shell FE model. This comparison is summarised in Table 2 which outlines that the maximum error remains
well below 1% for most of the members with few maximum differences of the order of 4%. Finally, the overall deflected shape of the arch bridge based on the loads considered in this study is proposed in Fig. 14.

Figure 12. Arch bridge.

Figure 13. Transverse distribution of the normal stresses of the concrete slab at different locations along the bridge length.
6. Conclusions

This paper presented a finite element formulation for the modelling of complex static schemes, such as those specified for arch, bow-string and cable-stayed bridges. For these particular bridge solutions, stress concentrations may be induced in the slab by the application of concentrated forces, i.e. due to the anchorage of prestressing cables or stays, or due to the presence of web members in arch bridges. The main advantage of the proposed formulation relies on its ability to retain the ease of use of a typical line element while being able to capture the complex response of the bridge deck along its length as well as across its width. This has been achieved as the proposed analytical and numerical model is derived based on partial interaction behaviour theory and is capable of capturing shear-lag effects. To highlight the ease of use of the proposed finite element two case studies have been considered for which stress concentrations significantly affect the structural response. For this purpose, a cable-stayed bridge and an arch bridge have been used. In both cases the results calculated using the proposed finite element have been compared against those obtained using a more refined model implemented using shell elements in a commercial finite element software and, in general, the results have been shown to be in good agreement. The limitation of the proposed formulation relies on its inability to account for torsion and transverse bending. The authors do not feel that this compromises the usefulness of the proposed element which intends to complement the analysis tools available to bridge designers. In fact, it could be used in preliminary design and costing, and as an efficient tool to determine effective width values for complex bridge systems.

Acknowledgment

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References


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Figure 14. Deflected shape of the arch bridge obtained using the proposed Deck FE model.
European Committee for Standardization.


