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Classification of Girders with I- or Box Cross-Sections

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Abstract

Steel structures may be currently designed accounting for inelastic material behaviour. However, this possibility is restricted by the occurrence of several instability modes. It is accordingly necessary to define requirements and restrictions to ensure a proper response in terms of ductility and strength related to the applicability of different methods of global analysis and member design. Modern steel design Codes impose such conditions in the form of a relevant classification. However, local buckling instability modes are usually accounted for separately, while lateral-torsional buckling is not considered at all in the classification process. Therefore the relevant rules refer to cross-sections rather than to members. In this paper the provisions for classification of some modern steel design Codes are reviewed. Experimental investigations that indicate the strong interaction between various instability modes are presented. An existing analytical model for the determination of the member behaviour including material and geometrical non-linear effects is briefly described. It is shown that its results compare well with corresponding test results. Extensive parametric studies are carried out for I- and box-girders and calibrated against test results. On this basis a new proposal for a possible classification of I- and box-girders is made. Initially, the cross-section is classified with due consideration of the local buckling interaction between the cross-section walls. This is followed for I-girders by setting slenderness limitations to lateral-torsional buckling in dependence on the method of global analysis and design.

Keywords: I- and box-girders, classification, local buckling, lateral-torsional buckling

1. Introduction

The response of steel framed structures is substantially influenced by a number of important effects, such as geometric and material nonlinear behaviour, imperfections, connection behaviour, etc. The application of appropriate tools allows for an inclusion of such effects in structural analysis. However, in practical design it is rather usual to perform simplified analysis, in which the influence of some effects is or is not taken into account, depending on its importance to the structural response. It is therefore necessary to set conditions in relation to the type of analysis that may be preformed and the effects that have to be taken into account. Such conditions are usually included in the design specifications and codes of practice in the form of classification in categories or classes. As an example, Eurocode 3 1992 includes rules for the classification of frames (braced/unbraced, sway/non-sway), connections (pinned, semi-rigid, rigid, full or partial strength), or cross-sections (classes 1 to 4) that serve certain design purposes and establish relations between local or overall structural response and the possible methods of analysis and member design.

Modern steel design codes include elastic or plastic methods of global analysis and member design. The combination of these methods leads to the distinction of 3 possible design procedures, which, according to the terminology of DIN 18800 1992, may be referred as '*elastic-elastic*', '*elastic-plastic*', and '*plastic-plastic*'. The application of these procedures is associated to specific requirements on the member behaviour. According to the first procedure, elastic methods are employed for both global analysis and member design. Members shall in this case be able to develop at least the yield moment of the cross-section. In the second procedure, global analysis is performed according to elastic methods but plastic methods are used for member design. Members shall therefore develop at least the plastic moment of their cross-section. Finally, the plastic-plastic procedure refers to the application of plastic methods for both global analysis and member design. The application of the method is associated to the provision of sufficient rotation capacity in the expected plastic zones, to allow for the necessary redistribution without drop in the bending resistance.

The response of the steel structural members is largely affected by the interaction between material and geometrical nonlinear effects. The geometrical nonlinearity results in several possible instability modes that may reduce substantially both the member resistance and its rotation capacity, thus canceling the positive influence of

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inelastic deformations and strain hardening effects. For steel girders that are examined here, such instability modes are local buckling of the cross-sectional walls (flanges and webs) and, in case of beams with I-sections, lateral-torsional buckling. It is possible to limit the detrimental influence of such instabilities by imposing limitations to the relevant slenderness parameters in relation to the expected member behaviour, i.e. to the method of analysis. With reference to the previously described design procedures, three classes of members may be introduced. As a result, structures with members complying with the limitations of the relevant classes may be designed according to the one or the other method.

As well known, when several instability modes occur, their possible interaction should be examined. It is for example expected, that in a cross section compact walls restrain against local buckling the more slender ones. In addition, lateral-torsional buckling may considerably reduce both the resistance and the rotation capacity of structural elements. However, current major steel design codes, and especially Eurocode 3 1992 examined here, usually account for various instability modes separately in the process of the classification. For example local buckling is examined separately for webs and flanges, ignoring the possible restraining affect of one to the other. Moreover, an influential parameter, the overall member slenderness in respect to lateral-torsional buckling, is not considered during the classification procedure, but only for member design. In such a way the violation of the limits set for the application of a certain design procedure due to a possible reduction in moment or rotation capacity cannot be taken into account. For example, the moment resistance of beams with class-1 cross-sections, according to Eurocode 3, exceeds the plastic moment of the section and the rotation capacity is sufficient to allow for fully redistribution of moments. However, if the beam is not adequately laterally supported the rotation capacity, or the moment resistance, may decrease so that the moment redistribution is not possible and the application of the plastic-plastic method of analysis not allowable.

In the present paper, a possible classification of beams with I- and box cross-sections is proposed, in which local buckling and lateral torsional buckling are examined. In this proposal local buckling is considered interactively for the cross-sectional walls, as well as lateral-torsional buckling for beams with I-sections. The classification refers therefore rather to the member than to the cross-section properties. For beams with box-sections the classification refers only to the cross-section properties, as such beams are usually not susceptible to lateral-torsional buckling.

After a short review of the provisions of some major steel design codes, a model for the determination of the member behaviour, including material and geometrical nonlinear effects, is presented. A statistical evaluation shows that the analytical results compare well with the results of similar experimental investigations. Subsequently extensive parametric studies are carried out in order to study the influence of the relevant slenderness parameters that influence the moment resistance and the rotation capacity of beams. Finally, appropriate expressions are derived that may serve as limits for classification.

2. Code provisions and recent studies

As previously stated, the application of certain procedures for global analysis and member design is associated to specific requirements on the performance of structural elements related to resistance and ductility. This performance is affected by local and global instability modes, as plate buckling of the cross-sectional walls and lateral-torsional buckling. The member susceptibility to such instability modes is reflected by the generalized slenderness parameter, according to

$$\bar{\lambda} = \sqrt{\frac{f_y}{\sigma_{cr}}},$$

where f_y is the yield strength and σ_{cr} the relevant buckling stress. With increasing slenderness the reductions in strength and ductility may lead to an inability of the members to develop the required properties for a specific type of analysis.

In the majority of steel design codes local and global instability modes are considered separately, leading to the consideration of the cross-section rather than the member. Furthermore, local buckling modes are treated independently, without taking into account any possible mutual restraining effects. Accordingly, following the notation of Eurocode 3, cross-section Classes 1 to 4 are introduced as following:

- Class-1 cross-sections shall possess sufficient rotation capacity at the level of the plastic moment. The application of the plastic-plastic design procedure requires the presence of class-1 sections at plastic hinges.
- Class-2 cross-sections shall have a moment resistance at least equal to the plastic moment of the section. The application of the elastic-plastic design procedure is then possible.
- Class-3 cross-sections shall have a moment resistance at least equal to the yield moment of the section, so that only the elastic-elastic design procedure may be applied.
- Class-4 cross-sections have a moment resistance less than the yield moment of the section. In such cases the elastic-elastic design procedure, in combination with the properties of the effective cross section, may be applied.

The limits of the different classes for welded I- and box-sections, according to the provisions of some major steel design codes (Eurocode 3 1992, DIN 18800 1990,

Table 1. Limiting width-to-thickness ratios for flanges and webs for classification of welded I cross-sections in bending

Design code	Class 1	Class 2	Class 3
Eurocode 3 Part 1.1	$c/t_f \le 9\varepsilon$ $d/t_w \le 72\varepsilon$	$c/t_f \le 10\varepsilon$ $d/t_w \le 83\varepsilon$	$c/t_f \le 14\varepsilon$ $d/t_w \le 124\varepsilon$
BS 5950 Part 1	$b/t_f \le 8\varepsilon$ $h_w/t_w \le 85\varepsilon$	$b/t_f \le 9\varepsilon$ $h_w/t_w \le 106\varepsilon$	$b/t_f \le 14\varepsilon$ $h_w/t_w \le 130\varepsilon$
DIN 18800 Part 1	$c/t_f \le 9\varepsilon$ $d/t_w \le 64\varepsilon$	$c/t_f \le 11\varepsilon$ $d/t_w \le 74\varepsilon$	$c/t_f \le 13\varepsilon$ $d/t_w \le 133\varepsilon$
AISC LRFD	$b/t_f \le 11\varepsilon$ $h_w/t_w \le 109\varepsilon$		$b/t_f \le 18\varepsilon/\sqrt{1-0.18\varepsilon^2}$ $h_w/t_w \le 165\varepsilon$
AIJ LSD	$\left(\frac{b/t_f}{13\varepsilon}\right)^2 + \left(\frac{h_w/t_w}{83\varepsilon}\right)^2 \le 1$	$\left(\frac{b/t_f}{13.5\varepsilon}\right)^2 + \left(\frac{h_w/t_w}{89\varepsilon}\right)^2 \le 1$	$\left(\frac{b/t_f}{15\varepsilon}\right)^2 + \left(\frac{h_w/t_w}{97\varepsilon}\right)^2 \le 1$

Table 2. Limiting width-to-thickness ratios for flanges and webs for classification of welded box cross-sections in bending

Design code	Class 1	Class 2	Class 3
Eurocode 3 Part 1.1	$b/t_f \le 33\varepsilon$ $d/t_w \le 72\varepsilon$	$b/t_f \le 38\varepsilon$ $d/t_w \le 83\varepsilon$	$b/t_f \le 42\varepsilon$ $d/t_w \le 124\varepsilon$
BS 5950 Part 1	$b/t_f \le 25\varepsilon$ $h_w/t_w \le 85\varepsilon$	$b/t_f \le 27\varepsilon$ $h_w/t_w \le 106\varepsilon$	$b/t_f \le 30\varepsilon$ $h_w/t_w \le 130\varepsilon$
DIN 18800 Part 1	$b/t_f \le 32\varepsilon$ $d/t_w \le 64\varepsilon$	$b/t_f \le 37\varepsilon$ $d/t_w \le 74\varepsilon$	$b/t_f \le 38\varepsilon$ $d/t_w \le 133\varepsilon$
AISC LRFD	$b/t_f \le 32\varepsilon$ $h_w/t_w \le 109\varepsilon$		$b/t_f \le 57\varepsilon$ $h_w/t_w \le 165\varepsilon$

BS 5950 1990, AISC LRFD 1994, AIJ LSD 1190 are illustrative summarized in Tables 1 and 2. The dimensions of the cross sections are shown in Fig. 1. The parameter ε is given by $\varepsilon = \sqrt{235/f_y}$, f_y being the yield strength of the steel in N/mm^2. It may be seen that the limits are examined independently for the webs and the flanges. The cross-section is classified according to its most unfavourable wall.

Additionally, web shear buckling is to be accounted for if the web slenderness ratio d/t_w exceeds certain limit (e.g. 65ε to 69ε according to various codes). Shear buckling effects are not considered in this paper.

An illustration of the tables is presented in Fig. 2. It may be noted that there are differences in the dimensions that the codes use to designate the relevant slenderness

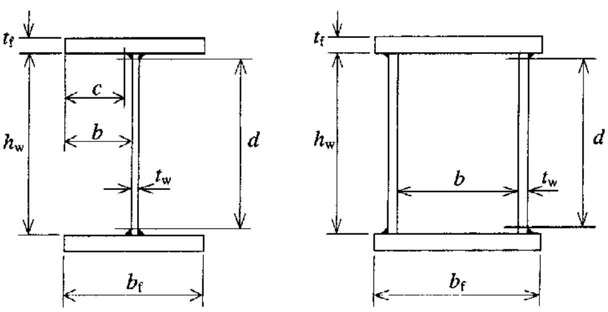


Figure 1. Notation of dimensions for I- and box cross-sections.

ratios. For example, for webs, Eurocode 3 and DIN 18800 use the clear depth d, while the other Codes use the depth h_w that includes the welds. Since this difference is usually small, no differentiation in this respect is made in Fig. 2. Generally, it may be stated that the provisions of BS 5950, DIN 18800 and AISC LRFD are qualitatively quite similar to those of Eurocode 3, with the exception that the latter does not distinguish between classes 1 and 2. That is the reason why it is represented by two curves, why the other Codes by three curves. However, quantitative differences between the Codes may be observed. For instance, BS 5950 is more conservative for flanges than Eurocode 3 and DIN 18800, especially in case of welded box sections. However, for webs this Code is more liberal. The provisions of AISC LRFD for webs appear more liberal compared to the other Codes. The rectangular shape of the limiting boundaries between the classes indicates that no interaction between flange and web buckling is accounted for. Only in the Japanese Code (AIJ LSD 1190) such an interaction is considered, leading to circular limiting curves.

Generally, the member resistance to lateral-torsional buckling has to be verified after the performance of the global analysis. As a result, the member resistance, or ductility, may be reduced below the limits of the stated requirements for a specific type of analysis. However, such an effect does generally not affect the proposed

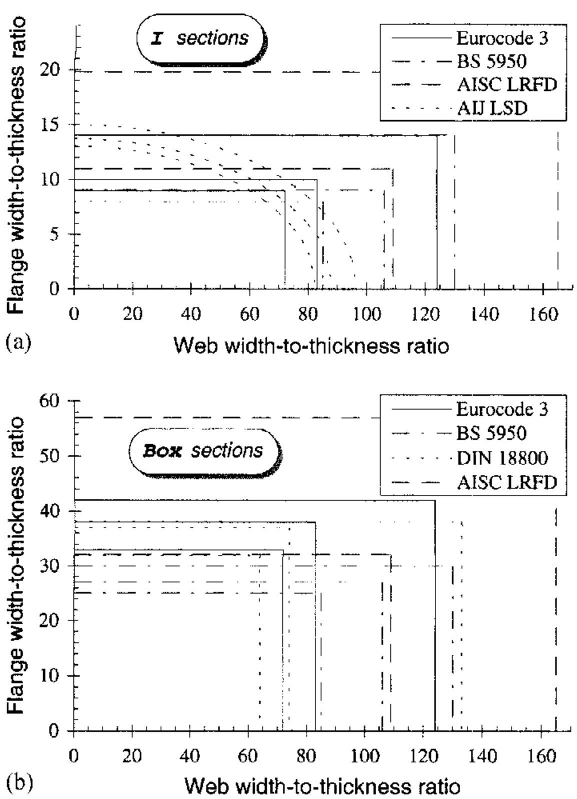


Figure 2. Boundaries for classification of cross-sections according to various codes.

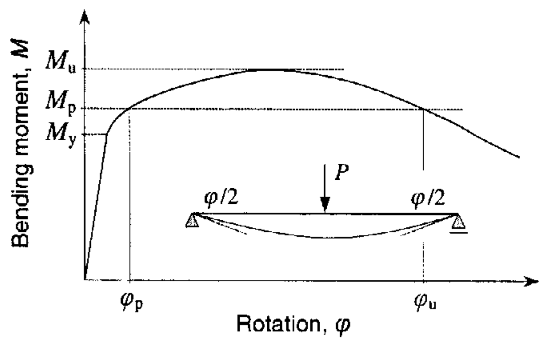


Figure 3. Three-point bending. Moment-rotation curve.

classification, and therefore the selection of the appropriate design procedure. DIN 18800 does not include any specific requirements for lateral bracing. Eurocode 3 states that when the plastic-plastic method is used, lateral bracing should be provided at plastic hinges, without any specification of the maximal distance. AISC LRFD goes further and proposes limiting values of the slenderness $\bar{\lambda}_{LT}$ against lateral-torsional buckling in case that the plastic moment is used for member design. Here again, only the Japanese Code (AIJ LSD 1190) includes lateral-torsional buckling in the classification procedure, setting limiting values of $\bar{\lambda}_{LT}$ for various classes. The member may then be classified in a particular class if both its cross-section and its slenderness for lateral-torsional buckling comply with the limits of this class. However, the limits for local and lateral-torsional buckling are considered independent to each other. The web resistance to shear buckling has also to be verified, especially for fabricated girders. However, shear-buckling effects are generally not included in the classification procedure, i.e. girders retain their class even if they are prone to shear buckling. Certainly, the interaction of shear buckling with the other instability modes for the purpose of classification, i.e. the selection of the analysis procedure, should be examined, especially for girders with high shear ratio. An appropriate model that includes shear buckling has been already proposed (Vayas 1998).

In some recent studies, Brune 1999 and 2000 derived limiting width-to-thickness ratios of webs and flanges for the different classes of cross sections. The study was based on the application of the effective width method, using a strain-oriented formulation for the plate slenderness, as proposed by Vayas and Psycharis 1992. Similar studies were made on I-girders made of high strength steel (Earls 1999). It was observed that a de-coupling of local and lateral-torsional buckling phenomena is not possible. Kemp 1986 proposed a model to account for the interaction between local flange, local web, and lateral-torsional buckling. Kemp 1996 analyzed test results and noted the difficulty in the derivation of a relationship between the available rotation capacity and the slenderness ratios of the cross-sectional walls when considered separately. He found the existence of a much better relationship between rotation capacity and a generalized slenderness, if the latter included the slenderness of both local and lateral-torsional buckling.

3. Experimental investigations

Numerous bending tests have been performed on girders with I cross-section, aimed primarily at defining the limits between classes 1 to 3. The majority of tests constitute three-point bending, where a simply supported beam is loaded at its mid-span (Fig. 3). The most important outcome from a test is the moment-rotation relationship, which provides the ultimate bending moment, M_u, and the rotation capacity of the beam. With the notation of Fig. 3, the rotation capacity R is defined as the ratio:

$$R = \frac{\varphi_u - \varphi_p}{\varphi_p}. \tag{1}$$

In the present paper, a total of 92 test results are considered (EC 3 Background document 1989), taken from the experimental investigations of Lukey & Adams 1969 (12 tests), Sawyer 1961 (21 tests), Kuhlmann 1989 (24 tests), Spangemacher & Sedlacek 1992 (29 tests) and Kemp 1986 (6 tests). The experimentally derived rotation capacity as a function of the width-to-thickness ratios of the web and the flange is presented in Fig. 4. The figure shows that there is no obvious relationship between the

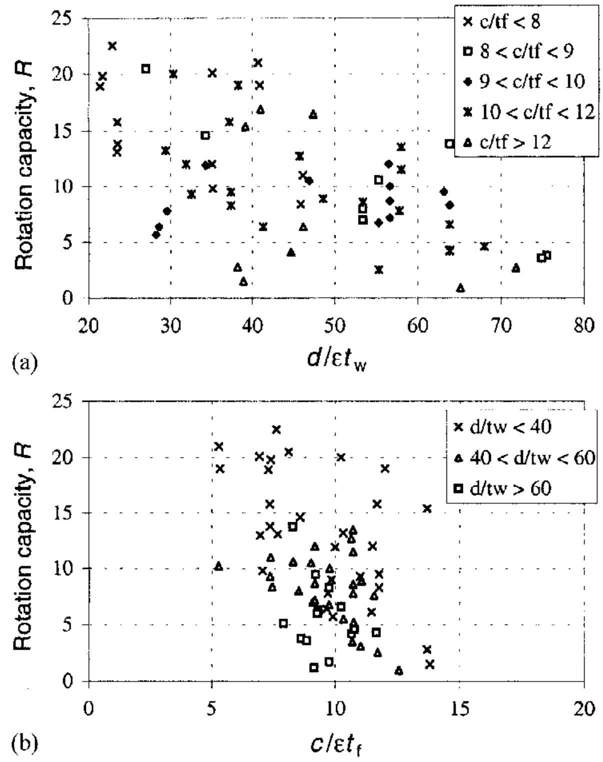
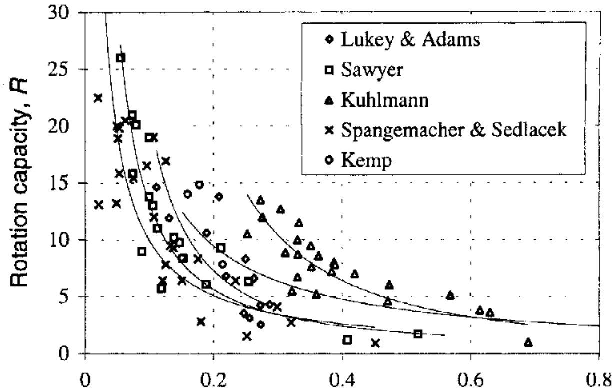


Figure 4. Rotation capacity of beams with I cross-sections in respect to web and flange slenderness as derived from tests.

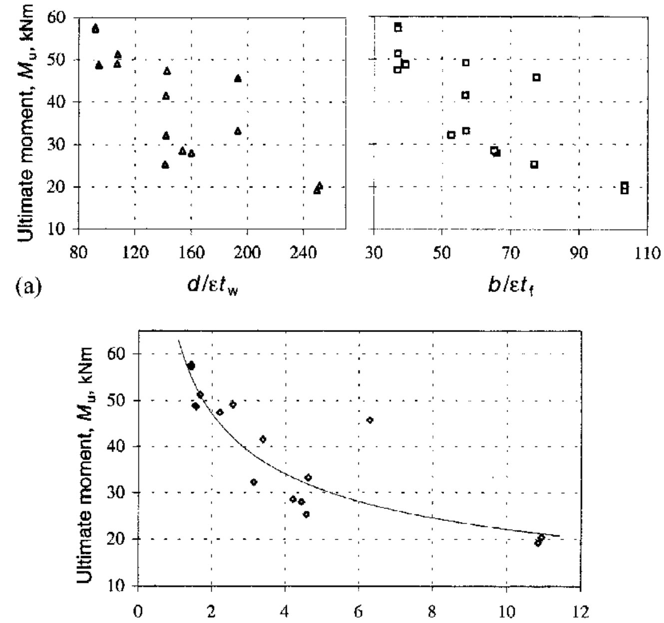


$\bar{\lambda}_{LT} \; \frac{c/(\varepsilon t_f)}{9} \; \frac{d/(\varepsilon t_w)}{72}$

Figure 5. Rotation capacity of beams with I cross-sections summary of test results.

rotation capacity, which serves as a limit between classes 1 and 2, and the slenderness of the cross-sectional walls, if considered separately. However, Fig. 5 shows a clear relationship between the rotation capacity and a generalized slenderness, in which the slenderness for local buckling of the compression walls and for lateral-torsional buckling are combined into a single parameter, demonstrating an interaction between the various buckling modes. The observed scatter in results may be explained by the difference in the realization of the supporting conditions, e.g. the end support against torsion, or the difference in the definition of the material properties, e.g. the value of the yield strength (static or lower yield point) of the material.

Similar experimental studies on box girders are rather scarce. In this paper, 17 four-point bending tests on simply supported girders loaded at the third of the span are considered (Scheer *et al.*, 1978). Fig. 6a shows the ten-



$\frac{b/(\varepsilon t_f)}{33} \; \frac{d/(\varepsilon t_w)}{72}$

Figure 6. Ultimate bending moment of box-girders vs (a) web and flange slenderness and (b) their product.

dency of the ultimate moment to decrease with increasing slenderness of the webs and the compressed flanges. However, a much clearer relationship between the moment capacity and the slenderness may be observed, if a generalized slenderness, combining the individual ones is used (Fig. 6b). In this case, the generalized slenderness does not include the slenderness for lateral-torsional buckling, as this does not constitute a failure mode due to the high torsional rigidity of the cross-section.

As a conclusion, it may be stated that test results on I- and box girders demonstrate the existing interaction between the possible local buckling modes and the coupling between local and lateral-torsional buckling for the case of I-girders.

4. Analytical model

The derivation of the bending resistance and the rotation capacity of steel beams has been the object of numerous analytical studies. The analysis tools range from the application of the finite element method, the consideration of plastic mechanisms, or the application of the effective width method, appropriately extended into the plastic region. An extensive reference list is given in Gioncu and Petcu 1997. In this paper the analysis is based on a analytical model, which is able to predict the member response for various stability problems (single plates, plate girders, stiffened plates, compression members) in the elastic and inelastic range. The various elements of the model and its applications are extensively described in several papers (Vayas and Psycharis 1992, 1993; Vayas 1996 and 1997; Vayas and Rangelov 1999). The model is also used and further developed for instability problems of members with hot-rolled, welded or cold-formed sections (Brune 1999, 2000; Wittemann 1993). Special attention was given to the agreement with test results, to which any numerical model shall be examined. It is therefore considered appropriate to give only a brief description of the model in the present paper and to focus on its experimental appraisal and its results for the stability problems regarded here.

Stability problems may be treated by definition of the relative slenderness as:

$$\bar{\lambda} = \sqrt{\frac{\sigma_0}{\sigma_{cr}}}, \tag{2}$$

where $\sigma_0 = E \cdot \varepsilon$ = stress corresponding to the constitutive law

ε = applied strain

σ_{cr} = critical buckling stress for the relevant instability mode

The nonlinear response and the influence of imperfections are accounted for by determination of a reduction factor χ(or ρ) according to the relevant buckling curve. The member response corresponding to the applied strain is then determined from:

$$\sigma = \chi \cdot \sigma_0 \tag{3}$$

The above methodology may be applied for the description of the element behaviour up to the yield stress/strain. The structural response beyond this limit may be described by appropriate extension of the above expressions into the inelastic range and thus beyond the limit load. This may be done by a strain-oriented definition of the slenderness for strains higher than the yield strain ε_y. Eq. (2) becomes then:

$$\bar{\lambda} = \sqrt{\frac{E\varepsilon}{\sigma_{cr}}}, \tag{4}$$

that may be written as:

$$\bar{\lambda} = \sqrt{\frac{E[\zeta\varepsilon_y + \eta(\varepsilon - \varepsilon_y)]}{\sigma_{cr}}}. \tag{5}$$

The parameters η and ζ are determined by theoretical considerations and comparisons with test results and are described in the relevant papers referred before. For girders examined here, the analytical procedure is carried out in three steps as following:

(i) *Cross-section response.* The response in terms of moment-curvature (M - κ) relationship is obtained by application of the above procedure to the component walls of the cross-section. For this purpose, the cross-section is decomposed in its individual plated elements (flanges, web). Assuming a linear strain distribution across the cross-section, the strain conditions in each plated element may be defined for each new value of the curvature. The corresponding slenderness for each individual plate is given by eq. (4) or (5), depending on whether the element strain is below or above the yield strain. The element response is determined by application of the Winter-curve, as each element behaves as a single plate. Finally the cross-section is composed again and its moment response to the applied curvature is found by integration of the relevant forces and moments of the individual plated elements. At weld positions rigid zones are introduced (Fig. 7), so that the width of individual plated elements is equal to the clear width between welds. The loading procedure is deformation controlled, i.e. the curvature is increased and the resulting moment response is determined, allowing thus the evaluation of the softening branch of the curve. The model is fully described and experimentally verified in previous papers (Vayas and Psycharis 1992, 1993; Vayas 1997).

(ii) *In-plane beam response.* The beam response is represented in terms of moment-rotation ($M - \varphi$) curves (Fig 3). In order to derive analytically such curves both prior to and after the attainment of the ultimate load, a strain-oriented procedure was

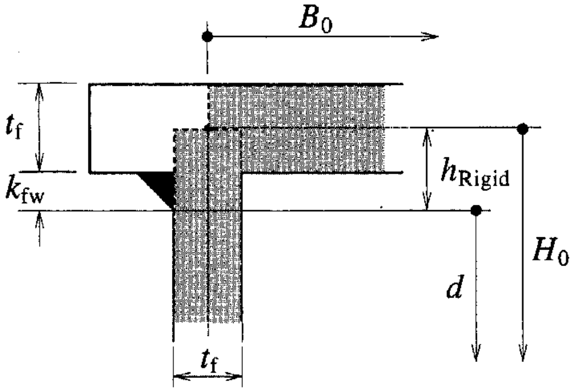


Figure 7. Rigid zones in box sections.

applied. Accordingly, a certain curvature at midspan κ_m is assumed that corresponds to a moment M_m at mid-span. For the three-point bending situation, the moments, and hence the curvatures, along the beam may be calculated. The end-rotation is then determined by appropriate integration, according to:

$$\varphi = \int_l M' \cdot \frac{M}{E \cdot I} dx = \int_l M' \cdot \kappa \cdot dx \qquad (6)$$

where κ is the distribution of the curvature along the beam and M' the virtual moments, having the value 1 at one support, 0 at the other support and a linear distribution along the bam.

The increase of κ_m beyond the value that corresponds to the moment capacity of the cross-section results in unloading in some portion of the beam. This portion corresponds to the length of the plastic zone and is assumed equal to 15% of the beam span. The model is fully described and experimentally verified for beams with both hot-rolled and cold-formed cross-section in previous papers (Vayas and Psycharis 1993, Vayas 1997, Wittemann 1993). The validation of the model for box-girders is shown later.

(iii) *Beam response including out-of-plane deformations.* The model for the inclusion of out-of plane deformations due to lateral-torsional buckling follows also a strain-oriented procedure. The slenderness due to lateral-torsional buckling is analogously redefined strain-oriented, in terms of curvature, as:

$$\bar{\lambda}_{LT0} = \sqrt{\frac{M}{M_{cr}}} = \sqrt{\frac{\kappa}{\kappa_{cr}}} \quad \text{for } \kappa \leq \kappa_{max} \qquad (6)$$

and

$$\bar{\lambda}_{LT0} = \sqrt{\frac{\kappa_{max} - (\kappa - \kappa_{max}) E_T / E}{\kappa_{cr}}} \quad \text{for } \kappa > \kappa_{max} \qquad (7)$$

In the above expression, $\kappa_{cr} = M_{cr}/EI$ is the curvature that corresponds to the elastic critical moment M_{cr} for lateral-torsional buckling. On this basis, the

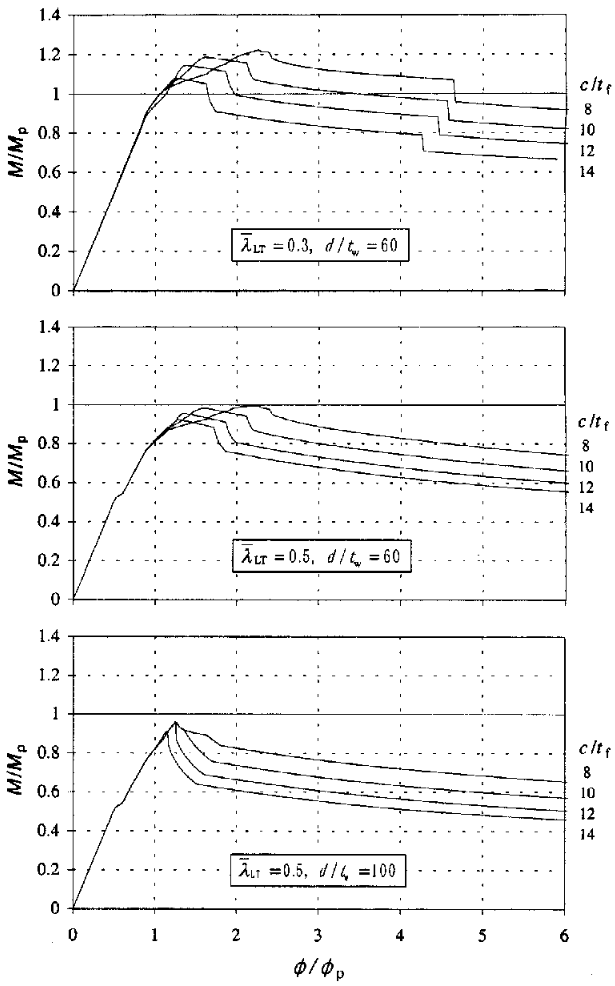


Figure 8. Momentrotation curves derived from the analysis of I-girders for different values of the main parameters.

already obtained moment-rotation curve for in-plane response is corrected to account for out-of-plane deformations (Vayas and Rangelov 1999). Typical moment-rotation curves for I-section girders with various slenderness values are presented in Fig. 8. Out-of plane deformations are considered only for I-girders. Box-girders, at least those with the usual dimensions, are due to their high torsional rigidity not prone to lateral-torsional buckling, so that only their in-plane response was studied.

The validation of the previously described model is done by comparison with experimental results for both I- and box-girders. The experimental investigations include 29 tests on I-girders (Spangemacher & Sedlacek 1992) and 17 tests on box-girders (Scheer *et al.*, 1978). The comparison between analytical model and test results shows for all girders an excellent correlation for both moment capacity and rotation capacity (Fig. 9). This demonstrates the appropriateness of the model (EC 3, Annex Z 1998). The correspondence of the mean values for the moment capacity is also excellent. The rotation capacity is under-estimated 25% on average. This is

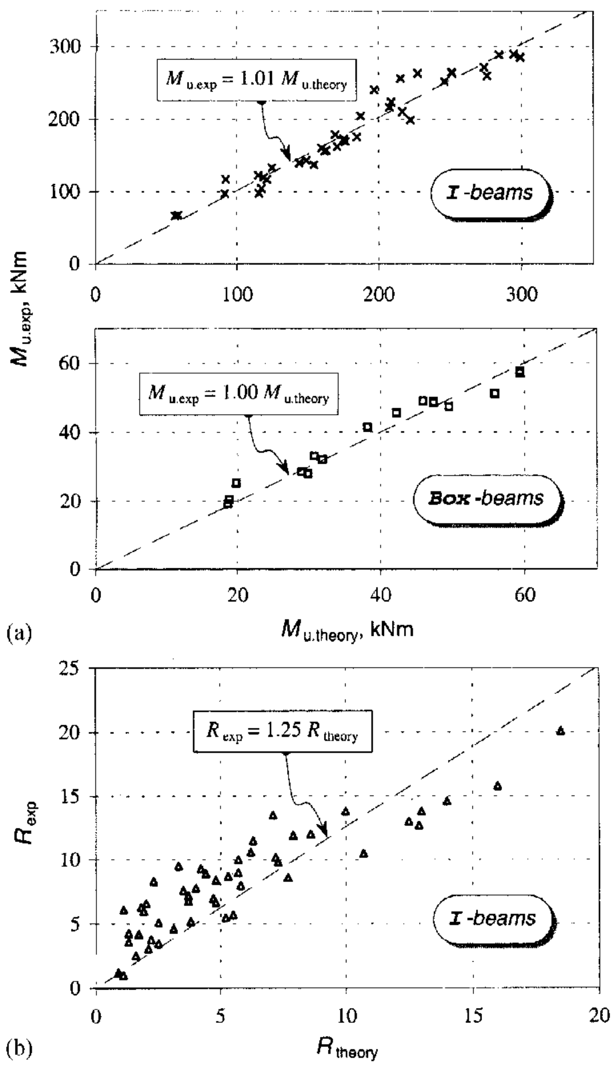


Figure. 9. Calibration of theoretical results against experimental ones in terms of (a) strength and (b) rotation capacity.

taken into account in the final proposal by appropriate corrections (EC 3, Annex Z 1998).

5. Parametric studies

Having the confidence of the model with experimental results, extensive parametric studies was carried out on girders with I- and box cross-sections. The following parameters have been considered to primarily influence the structural response:

- flange slenderness, λ_f, in the form of the limiting width-to-thickness ratios adopted by Eurocode 3:

$$\lambda_f=\begin{cases} c/\varepsilon t_f & for\ I-sections \\ b/\varepsilon t_f & for\ box-sections \end{cases} \tag{8}$$

- web slenderness, accordingly:

$$\lambda_w = d/\varepsilon t_w \tag{9}$$

- ratio k between the bending moment capacity of the cross-section composed of the flanges only, M_f, and the plastic bending resistance of the whole cross-section M_p:

$$k = M_f/M_p \tag{10}$$

- relative slenderness for lateral-torsional buckling defined as

$$\bar{\lambda}_{LT}=\sqrt{\frac{M_p}{M_{cr}}}. \tag{11}$$

To cover a wide range of possible practical applications, the values of the above parameters have been varied as listed in Table 3. Low-carbon steel S 235 has been considered with $\varepsilon = 1$.

Since the study is carried-out for the purpose of classification, the results are expressed in terms of rotation capacity R to distinguish between classes 1 and 2, the strength ratio M_u/M_p to establish the limit between classes 2 and 3, and the strength ratio M_u/M_y for the limit between classes 3 and 4. The results indicate that the ratio k, i.e. the distribution of material in the cross-section, has a small influence on both ductility and strength. Therefore fixed values of this parameter are considered, namely $k = 0.7$ for I-girders and $k = 0.6$ for box-girders.

The influence of the local buckling modes on the rotation capacity and on bending resistance is illustrated in Fig. 10 for I-girders, and in Fig. 11 for box-girders. Following observations can be made:

- Local buckling affects more the rotation capacity and less the bending resistance. The effects of the web and flange slenderness ratios are similar and of the same order (see Fig. 10(a) and Fig. 11(a)).
- The influence of the two slenderness ratios on the ultimate moment M_u appears to be not so significant, especially for λ_w and in the case of I-girders.
- Similarly to the tests results, the analytical results indicate a strong interaction in local buckling between the component walls of the cross-section.
- Lateral-torsional buckling for I-girders, as indicative shown in Fig. 12, appears to be very influential for

Table 3. Extend of parametric study

Parameter	λ_f	λ_w	k	$\bar{\lambda}_{LT}$
I-girders	8 10 12 14	60 70 80 90 100	0.6 0.7 0.8	0.3 0.4 0.5 0.65
Box-girders	25 30 35 40 45 50	55 60 70 80 100 125 140	0.5 0.6 0.7	-

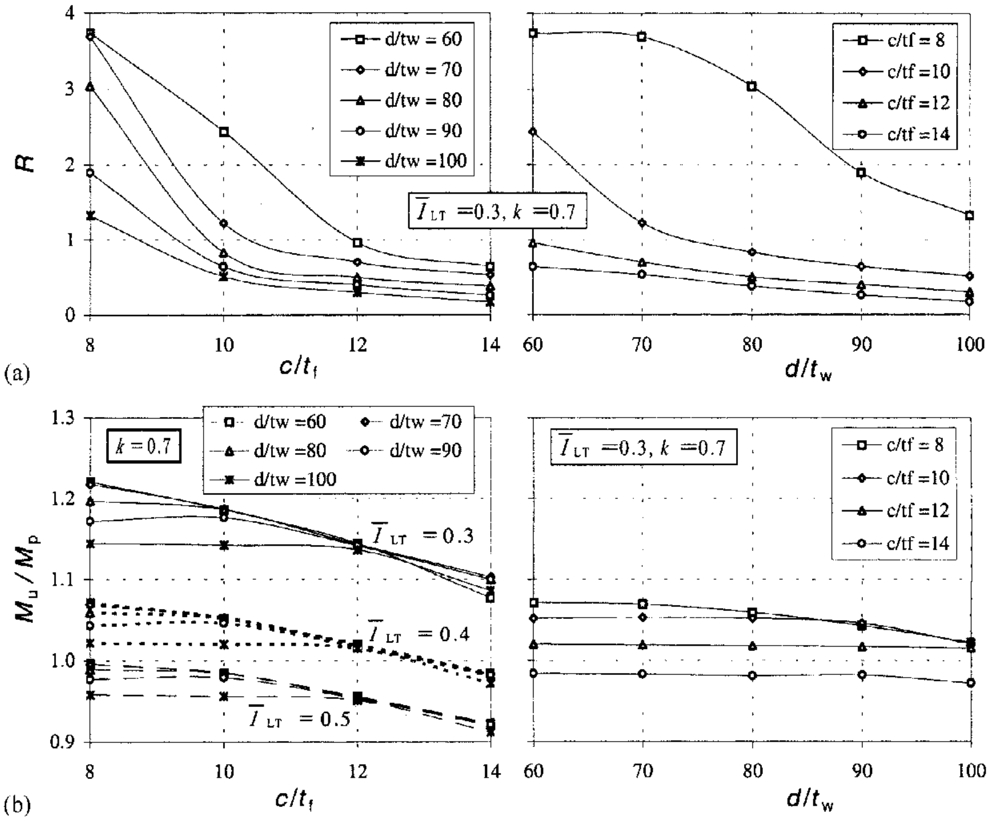


Figure 10. Influence of local buckling modes on (a) rotation capacity and (b) bending moment resistance of I-girders.

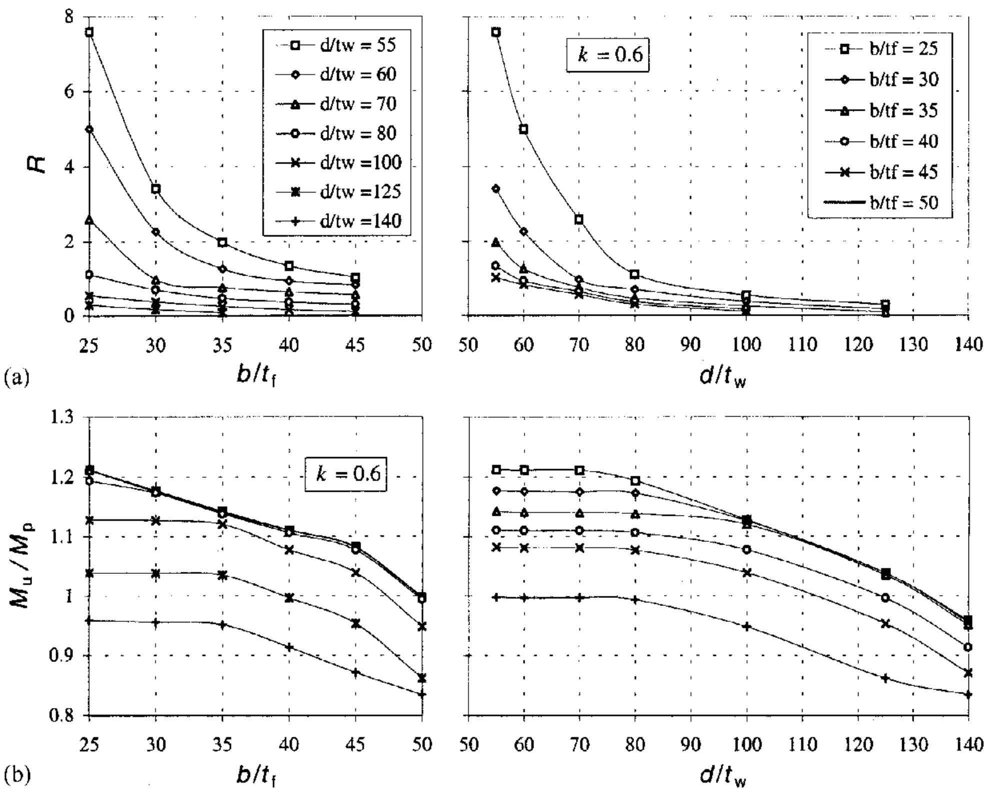


Figure 11. Influence of local buckling modes on (a) rotation capacity and (b) bending moment resistance of box-girders.

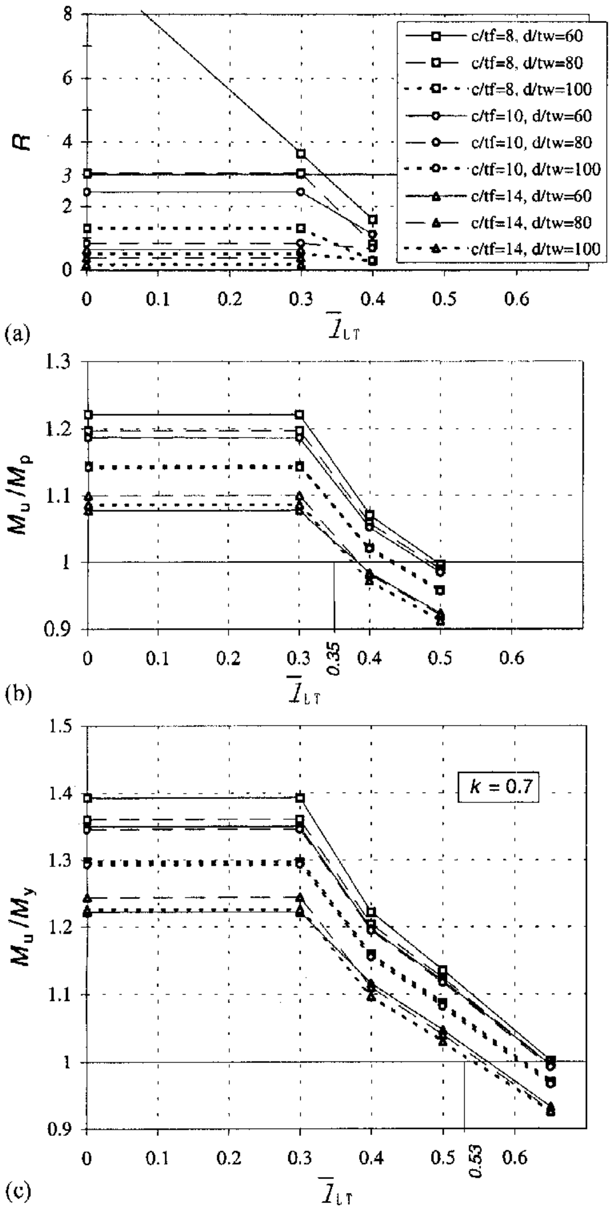


Figure 12. Influence of lateral-torsional buckling on (a) rotation capacity, (b) plastic bending moment resistance and (c) elastic bending moment resistance of I-girders.

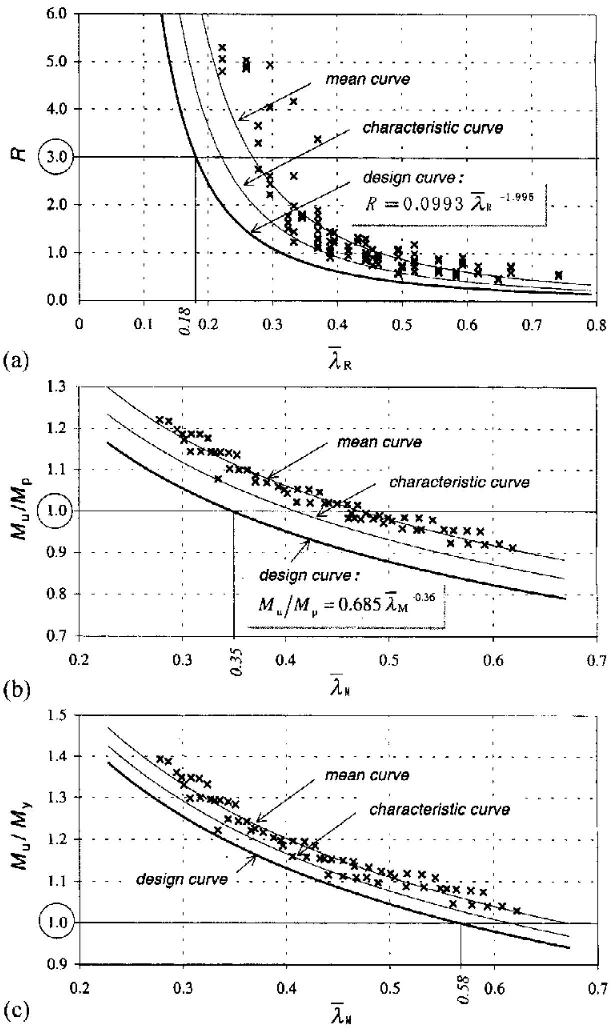


Figure 13. Design curves for classification of I-girders: (a) limit for class 1; (b) limit for class 2; (c) limit for class 3.

both rotation capacity and bending resistance. Its effect on the rotation capacity is especially high, since in very few cases with very compact walls adequate values of the rotation capacity were reached, even for $\bar{\lambda}_{LT} = 0.4$ that is the slenderness after which allowance should be made for lateral-torsional buckling (Eurocode 3 1992).

- Finally, the results shown in Fig. 12 indicate an interaction between lateral-torsional buckling and local buckling modes, which apparently deserves a due consideration in the classification of cross-sections and members.

6. Proposal for classification

6.1. Criteria for limitation between classes

To establish the limits between classes, the usual definitions adopted by the Codes will be used. As outlined before, the definitions of classes 2 and 3 are clear. In the former case the girder must develop the plastic moment, M_p, of the cross-section, in the latter the elastic (yield) moment, M_y. For the limit between classes 1 and 2 some discrepancy exists between the Codes in regard to the appropriate value of the rotation capacity. Eurocode 3 does not propose a certain value, stipulating that for Class 1 'sufficient' rotation capacity must be available to allow for a plastic redistribution of the bending moments. The AISC LRFD Specifications specify in the Commentary a value for the rotation capacity $R = 3$ for Class 1 sections, while the Japanese AIJ LSD Code $R = 4$. In this paper the former value is adopted. Accordingly, the criteria for the different classes are the following:

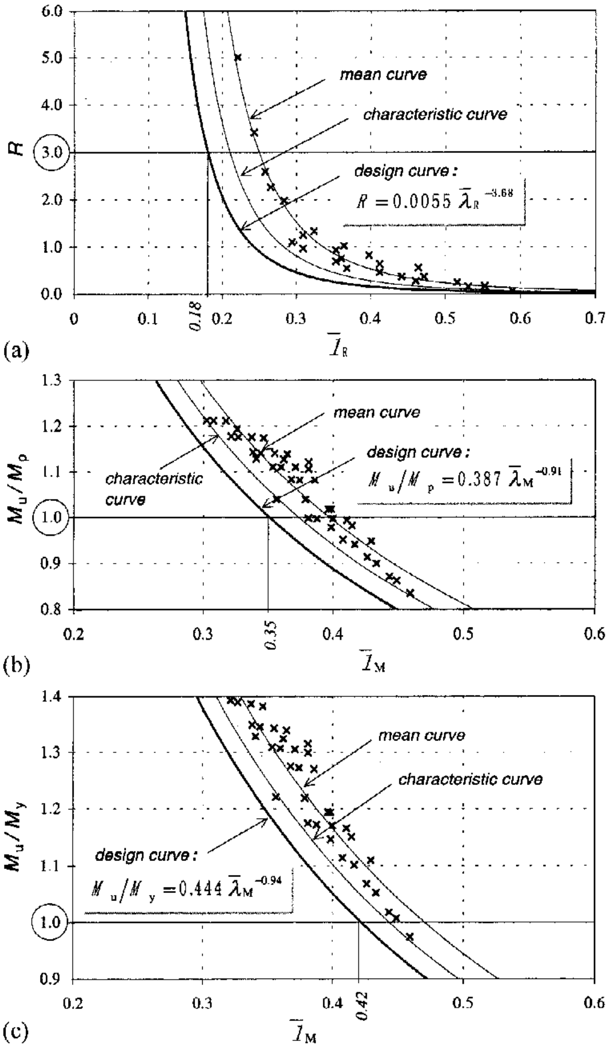


Figure 14. Design curves for classification of box-girders: (a) limit for class 1; (b) limit for class 2; (c) limit for class 3.

- for Class 1: $R \geq 3$
- for Class 2: $M_u/M_p \geq 1$
- for Class 3: $M_u/M_y \geq 1$

6.2. Generalized slenderness concept

To account for the interaction of the different buckling modes, a generalized slenderness is introduced. It includes the flange slenderness, λ_f, the web slenderness, λ_w, and, in the case of I-beams, the slenderness for lateral-torsional buckling $\bar{\lambda}_{LT}$. λ_f and λ_w are normalized by the limiting values for Class 1, λ_{f1} and λ_{w1}, respectively, as proposed by Eurocode 3.

Following the procedure of Eurocode 3, Annex Z 1998, several functions for the generalized slenderness have been examined. The following expressions have been proven as most appropriate:

- For I-girders:

$$\bar{\lambda}_R = \bar{\lambda}_{LT}\left(\frac{\lambda_f}{9}\right)\left(\frac{\lambda_w}{72}\right) \tag{12}$$

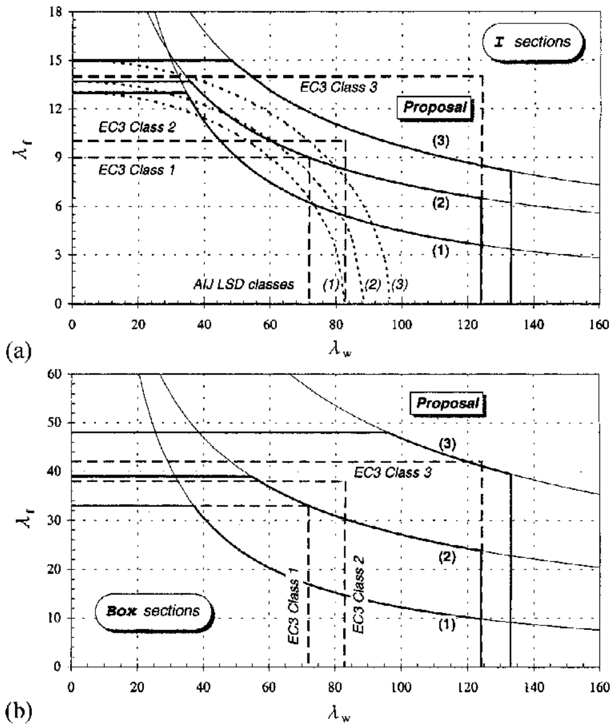


Figure 15. Proposed cross-section classification for (a) I-sections and (b) box-sections.

$$\bar{\lambda}_M = \bar{\lambda}_{LT}\left(\frac{\lambda_f}{9}\right)^{1/3}\left(\frac{\lambda_w}{72}\right)^{1/5} \tag{13}$$

- For box-girders:

$$\bar{\lambda}_R = 0.35\left(\frac{\lambda_f}{33}\right)\left(\frac{\lambda_w}{72}\right) \tag{14}$$

$$\bar{\lambda}_M = 0.35\left(\frac{\lambda_f}{33}\right)^{1/3}\left(\frac{\lambda_w}{72}\right)^{1/5} \tag{15}$$

It is interesting to observe that similar expressions for the powers of the local buckling slenderness were derived for both I- and for box-girders.

6.3. Classification of I- and box-cross-sections and girders

In order to estimate the limiting boundaries for the proposed classification, the results are presented in terms of the generalized slenderness in Figures 13 and 14 for I- and box-girders, respectively. The analytically derived rotation capacity of I-girders are increased by 25% to account for the difference in mean values to test results, as discussed in Section 4. Subsequently, a statistical evaluation, according to Eurocode 3, Annex Z 1998 was performed, which resulted in the derivation of mean, characteristic and design curves. The relevant limiting values for the slenderness are read directly from the design curves as illustrated in Figures 13 and 14.

Table 4. Proposal for classification of I- and box cross-sections

Cross-section	Class 1	Class 2	Class 3
I-section	$\bar{\lambda}_{LT} = 0.26$	$\bar{\lambda}_{LT} = 0.35$	$\bar{\lambda}_{LT} = 0.53$
	$\bar{\lambda}_R \leq 0.18$	$\bar{\lambda}_M \leq 0.35$	$\bar{\lambda}_M \leq 0.58$
	$\lambda_f \leq 13$	$\lambda_f \leq 13.5$	$\lambda_f \leq 15$
	$\lambda_w \leq 124$	$\lambda_w \leq 124$	$\lambda_w \leq 133$
Box-section	$\bar{\lambda}_R \leq 0.18$	$\bar{\lambda}_M \leq 0.35$	$\bar{\lambda}_M \leq 0.42$
	$\lambda_f \leq 33$	$\lambda_f \leq 39$	$\lambda_f \leq 48$
	$\lambda_w \leq 124$	$\lambda_w \leq 124$	$\lambda_w \leq 133$

Table 5. Proposal for classification of I-girders in respect to lateral-torsional buckling

Member Class 1	Member Class 2	Member Class 3
$\bar{\lambda}_M \leq 0.18$	$\bar{\lambda}_M \leq 0.35$	$\bar{\lambda}_M \leq 0.58$
$\bar{\lambda}_{LT} \leq 0.30$	$\bar{\lambda}_{LT} \leq 0.40$	$\bar{\lambda}_{LT} \leq 0.55$

In the classification of I-beams, lateral-torsional buckling must be accounted for in an explicit way. The classification is made accordingly in two stages. The cross-section is classified firstly without consideration of out-of-plane deformations. The corresponding limiting values of $\bar{\lambda}_{LT}$ are derived from Fig. 12. Fig. 12(a) shows that for Class 1 no allowance for lateral-torsional buckling is necessary if $\bar{\lambda}_{LT} \leq 0.30$. However, in this case, account must be also taken of the fact that, due to the strain hardening, the ultimate moment may increase well above the plastic moment (see Fig. 3). The ultimate moment in the girder for the development of the required rotation capacity is therefore higher than M_p. This over-strength is estimated to amount to approximately 30% (EC 3 Background document 1989). The value of $\bar{\lambda}_{LT}$ is then calculated according to eq. (11), taking into account the increased value of M_p. The corrected limiting value of $\bar{\lambda}_{LT}$ becomes then equal to $0.30/\sqrt{1.3} = 0.26$. The corresponding limits for Classes 2 and 3 are accordingly taken as 0.35 (Fig. 12(b)) and 0.53 (Fig. 12(c)), respectively. The classification of the cross-sections is then established on the basis of the limiting values of $\bar{\lambda}_R$ or $\bar{\lambda}_M$ obtained as the intersection points between the relevant criteria and the design curves, as shown in Figures 13 and 14, setting up $\bar{\lambda}_{LT}$ to the above limits.

The proposed classification is presented in Table 4 and illustrated in Fig. 15. The curved portions of the proposed boundaries correspond to the limitation in terms of $\bar{\lambda}_R$ or $\bar{\lambda}_M$, and represent explicitly the interaction between the component cross-section walls. Appropriate cut-off limits for λ_f and λ_w were adopted, indicating that the restraint supplied from a stocky to a slender wall is limited and that it is not possible to increase indefinitely the slenderness off wall by making the adjacent very stocky. The restraint is limited but considering a clamped situation for the very slender wall. It is noteworthy to point out that, when compared with the analytical results, the Eurocode 3 provisions are unsafe in some regions and conservative in others. In contrast, the proposed limiting relationships allow for certain class to use more slender webs if the flanges are very stocky and vice versa.

In a second stage, which is applicable only to the case of beams with I-sections, it is suggested to classify, as AIJ LSD 1190 does, not only the cross-section, but also the girder. The proposed member classification is summarized in Table 5. The limits for $\bar{\lambda}_{LT}$ are now slightly extended so that by increasing compactness of the cross-section the requirements for lateral-torsional buckling, i.e. for lateral support, may be relaxed. Thus, according to the proposal, an I-girder is classified into a certain class if (i) its cross-section satisfies the relevant requirements (Table 4), and (ii) the girder complies with the limitations given in Table 5. Finally, it should be recalled that allowance for shear buckling and for bending-shear interaction is necessary in the design process.

7. Conclusions

Based on theoretical investigations, duly calibrated against experimental results, a classification procedure for steel girders with I- and box cross-sections is proposed. Its purpose is to control the applicability of possible methods of analysis, i.e. how far inelastic material behaviour may be accounted for in global analysis and member design. The classification leads to slenderness limitations, or compactness requirements, since it is the appearance of instabilities that lowers the possibility to stress members beyond the elastic range. The limitations refer to the slenderness of the walls of the cross-sections and, for I-girders, the slenderness for lateral-torsional buckling.

The investigations lead to the following conclusions:

- Cross-sections should be classified according to the slenderness of their compressed walls to account for local buckling (as currently foreseen by most Codes)
- The interaction between local buckling of the individual walls of the cross-section (flanges, webs) should be accounted for within a certain region of slenderness (as currently foreseen by the Japanese Code)
- There is a need to limit the susceptibility to lateral-torsional buckling of I-girders by appropriate lateral bracing. The relevant requirements are dependent on the applied methods for global analysis and member design. This leads to limitations of the relevant slenderness in dependence to the above methods, and therefore to a corresponding classification (as currently foreseen by the Japanese Code but not the Eurocode 3)
- Based on the current analysis and after statistical evaluation with test results, appropriate classification rules are proposed in the present paper.

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